Equations and Relationships

MODULE 11

CALIFORNIA

ESSENTIAL QUESTION

How can you use equations and relationships to solve real-world problems?

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Real-World Video

Businesses use equations to determine how many products can be made from the materials available. A guitar company uses an equation to determine how many guitars can be made with x amount of strings.

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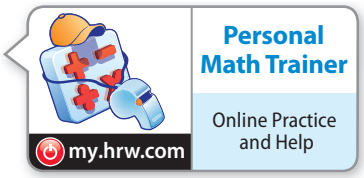
Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Evaluate Expressions

EXAMPLE Evaluate $8(3+2) - 5^2$

$8(3+2) - 5^2 = 8(5) - 5^2$ Perform operations inside parentheses first.

$= 8(5) - 25$ Evaluate exponents.

$= 40 - 25$ Multiply.

$= 15$ Subtract.

Evaluate the expression.

1. $4(5 + 6) - 15$ _____

2. $8(2 + 4) + 16$ _____

3. $3(14 - 7) - 16$ _____

4. $6(8 - 3) + 3(7 - 4)$ _____

5. $10(6 - 5) - 3(9 - 6)$ _____

6. $7(4 + 5 + 2) - 6(3 + 5)$ _____

7. $2(8 + 3) + 4^2$ _____

8. $7(14 - 8) - 6^2$ _____

9. $8(2 + 1)^2 - 4^2$ _____

Connect Words and Equations

EXAMPLE The product of a number and 4 is 32.

The product of x and 4 is 32. Represent the unknown with a variable.

$4 \times x$ is 32. Determine the operation.

$4 \times x = 32$. Determine the placement of the equal sign.

Write an algebraic equation for the word sentence.

10. A number increased by 7.9 is 8.3. _____

11. 17 is the sum of a number and 6. _____

12. The quotient of a number and 8 is 4. _____

13. 81 is three times a number. _____

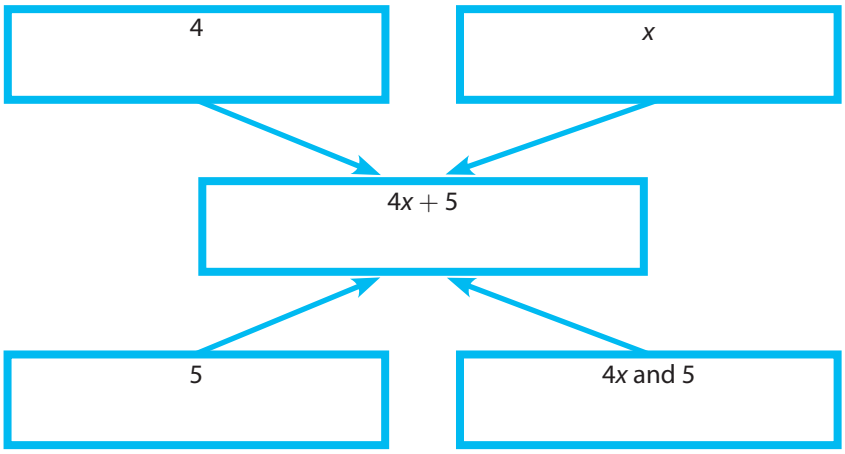
14. The difference between 31 and a number is 7. _____

15. Eight less than a number is 19. _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.



Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. algebraic expression	**A.** A mathematical statement that two expressions are equal.
2. equation	**B.** A value of the variable that makes the statement true.
3. solution	**C.** A mathematical statement that includes one or more variables.

Vocabulary

Review Words

- ✔ algebraic expression *(expresión algebraica)*
- ✔ coefficient *(coeficiente)*
- ✔ constant *(constante)*
- evaluating *(evaluar)*
- like terms *(términos semejantes)*
- ✔ term *(término, en una expresión)*
- ✔ variable *(variable)*

Preview Words

- equation *(ecuación)*
- equivalent expression *(expresión equivalente)*
- properties of operations *(propiedades de las operaciones)*
- solution *(solución)*

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

GETTING READY FOR

Equations and Relationships

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

CA CC 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Key Vocabulary

equation *(ecuación)*
A mathematical sentence that shows that two expressions are equivalent.

What It Means to You

You will learn to write an equation to represent a situation.

EXAMPLE 6.EE.7

The Falcons won their football game with a score of 30 to 19. Kevin scored 12 points for the Falcons. Write an equation to determine how many points Kevin's teammates scored.

Kevin's points	+	Teammates' points	=	Total points
12	+	t	=	30

CA CC 6.EE.5

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Key Vocabulary

inequality *(desigualdad)*
A mathematical sentence that shows the relationship between quantities that are not equal.

What It Means to You

You can substitute a given value for the variable in an equation or inequality to check if that value makes the equation or inequality true.

EXAMPLE 6.EE.5

Melanie has to buy 6 tickets to a play. She will pay at least $156 depending on the price of the seats. Write an inequality for this situation. Use it to decide if $20 and $30 are possible ticket prices.

Number of tickets bought	·	Price per ticket	≥	Total cost
6	·	p	≥	156

Substitute 20 and 30 for p to see if the inequality is true.

$6p \geq 156$

$6 \cdot 20 \stackrel{?}{\geq} 156$

$120 \stackrel{?}{\geq} 156$ ✗

$6p \geq 156$

$6 \cdot 30 \stackrel{?}{\geq} 156$

$180 \stackrel{?}{\geq} 156$ ✓

The price per ticket could be $30 but not $20.

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LESSON 11.1

Writing Equations to Represent Situations

CA CC 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *Also 6.EE.5, 6.EE.6*

ESSENTIAL QUESTION

How do you write equations and determine whether a number is a solution of an equation?

Determining Whether Values Are Solutions

An **equation** is a mathematical statement that two *expressions* are equal. An equation may or may not contain variables. For an equation that contains a variable, a **solution** of the equation is a value of the variable that makes the equation true.

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An expression represents a single value.

An equation represents a relationship between two values.

	Expression	Equation
Numerical	$5 + 4$	$5 + 4 = 9$
Words	a number *plus* 4	A number *plus* 4 *is* 9.
Algebraic	$n + 4$	$n + 4 = 9$

An equation relates two expressions using a symbol for "is" or "equals."

EXAMPLE 1

CA CC 6.EE.5

Determine whether the given value is a solution of the equation.

A $x + 9 = 15; x = 6$

$6 + 9 \stackrel{?}{=} 15$ Substitute 6 for x.

$15 \stackrel{?}{=} 15$ Add.

6 is a solution of $x + 9 = 15$.

B $\frac{y}{4} = 32; y = 8$

$\frac{8}{4} \stackrel{?}{=} 32$ Substitute 8 for y.

$2 \stackrel{?}{=} 32$ Divide.

8 is not a solution of the equation $\frac{y}{4} = 32$.

C $8x = 72; x = 9$

$8(9) \stackrel{?}{=} 72$ Substitute 9 for x.

$72 \stackrel{?}{=} 72$ Multiply.

9 is a solution of $8x = 72$.

Math Talk
Mathematical Practices

Does the variable in each equation in parts A–D refer to a single number? Explain.

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YOUR TURN

Determine whether the given value is a solution of the equation.

1. $11 = n + 6; n = 5$

2. $y - 6 = 24; y = 18$

3. $\frac{x}{9} = 4; x = 36$

4. $15t = 100; t = 6$

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Writing Equations to Represent Situations

You can represent some real-world situations with an equation. Making a model first can help you organize the information and identify what the variable represents.

EXAMPLE 2

Real World

CA CC 6.EE.6

Mark scored 17 points for the home team in a basketball game. His teammates as a group scored *p* points. Write an equation to represent this situation.

Mark's points	+	Teammates' points	=	Total points
↓		↓		↓
17	+	*p*	=	46

p represents the number of points Mark's teammates scored.

YOUR TURN

Write an equation to represent each situation.

5. The sum of a number *f* and 9 is equal to 38.

6. The product of 17 and a number *n* is equal to 102.

7. Craig is *c* years old. His 12-year-old friend Kaitlin is $\frac{1}{2}$ year younger than Craig.

8. Kim rented skates for *h* hours. The rental fee was $2.50 per hour. She paid a total of $20.

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Writing an Equation and Checking Solutions

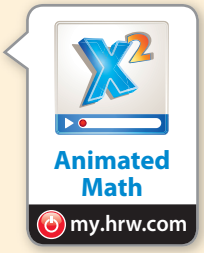
You can substitute a given value for the variable in a real-world equation to check if that value makes sense for the situation.

EXAMPLE 3

Real World

CA CC 6.EE.7

Sarah used a gift card to buy $47 worth of food. She has $18 left on her gift card. Write an equation to represent this situation, and use it to determine whether Sarah had $65 or $59 on the gift card before buying food.



STEP 1 Write a word equation based on the situation.

Amount on card	–	Amount spent	=	Amount left on card

STEP 2 Rewrite the equation using a variable for the unknown quantity and the given values for the known quantities.

Let x be the amount on the card.

Amount on card	–	Amount spent	=	Amount left on card
x	–	47	=	18

The amount spent and the amount left on the card are the known quantities. Substitute those values in the equation.

STEP 3 Substitute 65 and 59 for x to see which equation is true.

$$x - 47 = 18 \qquad x - 47 = 18$$
$$65 - 47 \stackrel{?}{=} 18 \qquad 59 - 47 \stackrel{?}{=} 18$$
$$18 \stackrel{?}{=} 18 \qquad 12 \stackrel{?}{=} 18$$

The amount on Sarah's gift card before she bought food was $65.

Reflect

9. **What If?** Suppose Sarah had $12 left on her gift card. How would this change the equation and the final answer?

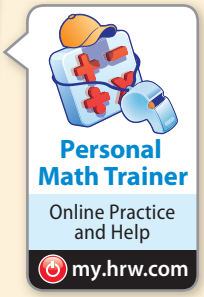
_____

YOUR TURN

10. On Saturday morning, Owen earned $24. By the end of the afternoon he had earned a total of $62. Write an equation to represent the situation. Tell what the variable represents. Determine whether Owen earned $38 or $31 on Saturday afternoon.

_____

_____



Guided Practice

Determine whether the given value is a solution of the equation. (Example 1)

1. $23 = x - 9; x = 14$ _____

2. $\frac{n}{13} = 4; n = 52$ _____

3. $25 = \frac{k}{5}; k = 5$ _____

4. $2.5n = 45; n = 18$ _____

5. $21 - h = 15; h = 6$ _____

6. $d - 4 = 19; d = 15$ _____

7. $w - 9 = 0; w = 9$ _____

8. $5q = 31; q = 13$ _____

9. Each floor of a hotel has r rooms. On 8 floors, there are a total of 256 rooms. Write an equation to represent this situation. (Example 2)

Number		Number of rooms			
_____	×	_____	=	_____	_____

In Exercises 10–12, write an equation to represent the situation. (Example 2)

10. When a number n is divided by 16, the result is 3. _____

11. The sum of 4.5 and a number k is 26.2. _____

12. In the school band, there are 5 trumpet players and f flute players. There are twice as many flute players as there are trumpet players. _____

In Exercises 13 –14, write an equation to solve the problem. Tell what your variable represents. (Example 3)

13. Pedro bought 8 tickets to a basketball game. He paid a total of $208. Determine whether each ticket cost $26 or $28.

_____

14. The high temperature was 92 °F. This was 24 °F higher than the overnight low temperature. Determine whether the low temperature was 62 °F or 68 °F.

_____

_____

ESSENTIAL QUESTION CHECK-IN

15. Tell how you can determine whether a number is a solution of an equation.

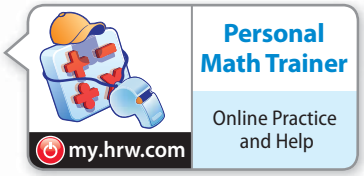
_____

_____

Name _____ Class _____ Date _____

11.1 Independent Practice

CA CC 6.EE.5, 6.EE.6, 6.EE.7



In Exercises 16–18, write an equation to solve the problem. Tell what your variable represents.

16. Andy's grandfather, who is 76 years old, is 4 times as old as Andy. Determine whether Andy is 19 or 22 years old.

17. Mi's sleeping bag weighs 8 pounds. Her backpack and sleeping bag together weigh 31 pounds. Determine whether her backpack alone weighs 25 or 23 pounds.

18. Halfway through a bus route, 23 students have been dropped off and 48 students remain on the bus. Determine whether there were 61 or 71 students on the bus to begin with.

19. Write an equation that involves multiplication, contains a variable, and has a solution of 5. Then write another equation that has the same solution and includes the same variable and numbers but uses division.

20. **Vocabulary** How are expressions and equations different?

21. **Multistep** Alan has partially completed a table showing the distances between his town, Greenville, and two other towns.

Distance between Greenville and Nearby Towns (miles)	
Parker	29
Hadley	?

a. The distance between Hadley and Greenville is $12\frac{1}{2}$ miles less than the distance between Parker and Greenville. Write two equations that compare the distance between Hadley and Greenville and the distance between Parker and Greenville. Tell what your variable represents.

b. Alan says the distance from Hadley to Greenville is $16\frac{1}{2}$ miles. Is he correct? Explain.

22. **Explain the Error** A problem states that Ursula earns $9 per hour. To write an expression that tells how much money Ursula earns for h hours, Joshua wrote $\frac{9}{h}$. Sarah wrote $9h$. Whose expression is correct and why?

23. **Communicate Mathematical Ideas** A dog weighs 44 pounds. The dog needs to lose 7 pounds to reach its ideal weight. Mikala wrote the equation $x + 7 = 44$ to represent the situation. Kirk wrote the equation $44 - x = 7$. What does the variable represent? Which equation is correct? Can you write another equation that represents the situation?

24. **Multiple Representations** The table shows the ages of Cindy and her dad.

Dad's Age	Cindy's Age
28 years old	2 years old
36 years old	10 years old
?	18 years old

a. Write an equation that relates Cindy's age to her dad's age when Cindy is 18. Tell what the variable represents.

b. Determine if 42 is a solution to the equation. Show your work.

c. Explain the meaning of your answer in part **b**.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

25. **Critical Thinking** In the school band, there are 4 trumpet players and f flute players. The total number of trumpet and flute players is 12. Are there twice as many flute players as trumpet players? Explain.

26. **Problem Solving** Ronald paid $162 for 6 tickets to a basketball game. During the game he noticed that his friend paid $130 for 5 tickets. The price of each ticket was $26. Was Ronald overcharged? Explain.

27. **Communicate Mathematical Ideas** Tariq said you can write an equation by setting an expression equal to itself. Would an equation like this be true? Explain.

LESSON 11.2

Addition and Subtraction Equations

CA CC 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *Also 6.EE.5, 6.EE.6*

ESSENTIAL QUESTION

How do you solve equations that contain addition or subtraction?

EXPLORE ACTIVITY Real World

CA CC 6.EE.6, 6.EE.7

Modeling Equations

A puppy weighed 6 ounces at birth. After two weeks, the puppy weighed 14 ounces. How much weight did the puppy gain?

Let x represent the number of ounces gained.

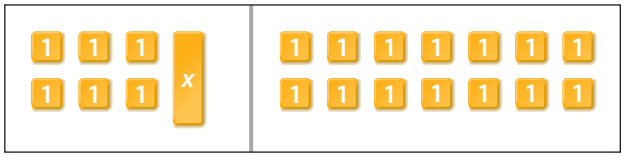
Weight at birth	+	Weight gained	=	Weight after 2 weeks
↓		↓		↓
6	+	x	=	14

To answer this question, you can solve the equation $6 + x = 14$.

Algebra tiles can model some equations. An equation mat represents the two sides of an equation. To solve the equation, remove the same number of tiles from both sides of the mat until the x tile is by itself on one side.

A Model $6 + x = 14$.

B How many 1 tiles must you remove on the left side so that the x tile is by itself? _____
Cross out these tiles on the equation mat.



$6 + x$ 14

C Whenever you remove tiles from one side of the mat, you must remove the same number of tiles from the other side of the mat. Cross out the tiles that should be removed on the right side of the mat.

D How many tiles remain on the right side of the mat? _____
This is the solution of the equation.

The puppy gained _____ ounces.

Math Talk
Mathematical Practices
Why did you remove tiles from each side of your model?

Reflect

1. **Communicate Mathematical Ideas** How do you know when the model shows the final solution? How do you read the solution?

_____

_____

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Using Subtraction to Solve Equations

Removing the same number of tiles from each side of an equation mat models subtracting the same number from both sides of an equation.

Subtraction Property of Equality

You can subtract the same number from both sides of an equation, and the two sides will remain equal.

When an equation contains addition, solve by subtracting the same number from both sides.

EXAMPLE 1

CA CC 6.EE.5

Solve the equation $a + 15 = 26$. Graph the solution on a number line.

$a + 15 = 26$ Notice that the number 15 is added to a.

$$\begin{array}{rl} a + 15 = & 26 \\ -15 \quad & -15 \\ \hline a \quad = & 11 \end{array}$$

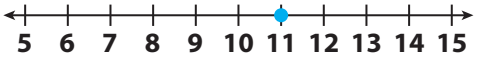
Subtract 15 from both sides of the equation.

Check: $a + 15 = 26$

$11 + 15 \stackrel{?}{=} 26$ Substitute 11 for a.

$26 \stackrel{?}{=} 26$ Add on the left side.

Graph the solution on a number line.



Reflect

2. **Communicate Mathematical Ideas** How do you decide which number to subtract from both sides?

_____

_____

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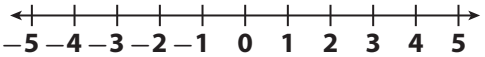
YOUR TURN

3. Solve the equation $5 = w + 1\frac{1}{2}$.

Graph the solution on a number line.



$w =$ _____

Using Addition to Solve Equations

When an equation contains subtraction, solve by adding the same number to both sides.

Addition Property of Equality

You can add the same number to both sides of an equation, and the two sides will remain equal.

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EXAMPLE 2

CA CC 6.EE.5

Solve the equation $y - 21 = 18$. Graph the solution on a number line.

$y - 21 = 18$ Notice that the number 21 is subtracted from y.

$$\begin{array}{rr} y - 21 = & 18 \\ +\ 21 & +21 \\ \hline y = & 39 \end{array}$$

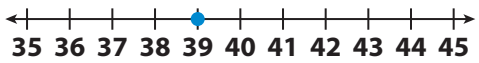
Add 21 to both sides of the equation.

Check: $y - 21 = 18$

$39 - 21 \stackrel{?}{=} 18$ Substitute 39 for y.

$18 \stackrel{?}{=} 18$ Subtract.

Graph the solution on a number line.



Reflect

4. **Communicate Mathematical Ideas** How do you know whether to add on both sides or subtract on both sides when solving an equation?

_____

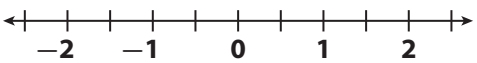
_____

_____

My Notes

YOUR TURN

5. Solve the equation $h - \frac{1}{2} = \frac{3}{4}$.



Graph the solution on a number line.

$h =$ _____

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Solving Equations that Represent Geometric Concepts

You can write equations to represent geometric relationships.

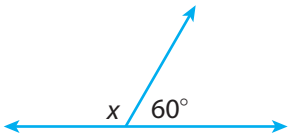
Recall that a straight line has an angle measure of 180°. Two angles whose measures have a sum of 180° are called supplementary angles. Two angles whose measures have a sum of 90° are called complementary angles.

EXAMPLE 3

CA CC 6.EE.7, 6.EE.6

Find the measure of the unknown angle.

STEP 1 Write the information in the boxes.



Unknown angle measure	+	60°	=	180°

STEP 2 Write a description to represent the model. Include a question for the unknown angle measure.

The sum of the measure of an unknown angle and 60° is 180°. What is the measure of the unknown angle?

STEP 3 Write an equation.

$x + 60 = 180$

x represents the measure of the unknown angle in degrees.

STEP 4 Solve the equation.

$$\begin{array}{rrcr} x + & 60 & = & 180 \\ & \underline{-60} & & \underline{-60} \\ x & & = & 120 \end{array}$$

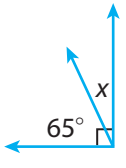
Subtract 60 from each side.

The unknown angle measures 120°.

The final answer includes units of degrees.

YOUR TURN

6. Write and solve an equation to find the measure of the unknown angle. Tell what the variable represents.



_____

7. Write and solve an equation to find the complement of a 42° angle. Tell what the variable represents.

_____

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Writing Real-World Problems for a Given Equation

You can write a real-world problem for a given equation. Examine each number and mathematical operation in the equation.

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EXAMPLE 4

Real World

CA CC 6.EE.7

Write a real-world problem for the equation $21.79 + x = 25$. Then solve the equation.

STEP 1 Examine each part of the equation.

x is the unknown or quantity we are looking for.

21.79 is added to x.

$= 25$ means that after adding 21.79 and x, the result is 25.

STEP 2 Write a real-world situation that involves *adding* two quantities.

Joshua wants to buy his mother flowers and a card for Mother's Day. Joshua has $25 to spend and selects roses for $21.79. How much can he spend on a card? Let x represent that amount.

Math Talk
Mathematical Practices
How is the question in a real-world problem related to its equation?

STEP 3 Solve the equation.

$$\begin{array}{rcr} 21.79 + x = & & 25 \\ \underline{-21.79} & & \underline{-21.79} \\ x = & & 3.21 \end{array}$$

The final answer includes units of money in dollars.

Joshua can spend $3.21 on a Mother's Day card.

Reflect

8. **What If?** How might the real-world problem change if the equation were $x - 21.79 = 25$ and you still used roses for $21.79?

_____

_____

_____

YOUR TURN

9. Write a real-world problem for the equation $x - 100 = 40$. Tell what the variable represents. Then solve the equation.

_____

_____

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Guided Practice

1. A total of 14 guests attended a birthday party. Three friends stayed after the party to help clean up. How many left when the party ended? (Explore Activity)

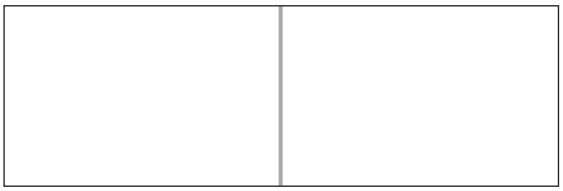
a. Let x represent the _____

b.

Number that	+	Number that	=	

_____ + _____ = _____

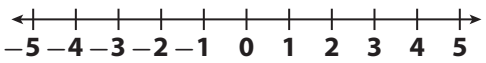
c. Draw algebra tiles to model the equation.



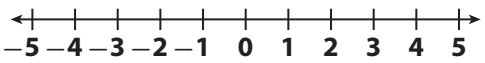
_____ friends left when the party ended.

Solve each equation. Graph the solution on a number line. (Examples 1 and 2)

2. $2 = x - 3$ $x =$ _____



3. $s + 12.5 = 14$ $s =$ _____



Solve each equation. (Examples 1 and 2)

4. $h + 6.9 = 11.4$

$h =$ _____

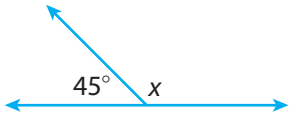
5. $82 + p = 122$

$p =$ _____

6. $n + \frac{1}{2} = \frac{7}{4}$

$n =$ _____

7. Write and solve an equation to find the measure of the unknown angle. Tell what the variable represents. (Example 3)



_____

8. Write a real-world problem for the equation $x - 75 = 200$. Tell what the variable represents. Then solve the equation. (Example 4)

_____

ESSENTIAL QUESTION CHECK-IN

9. How do you solve equations that contain addition or subtraction?

_____

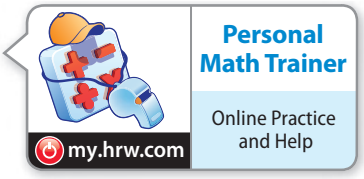
_____

_____

Name _____ Class _____ Date _____

11.2 Independent Practice

CA CC 6.EE.5, 6.EE.6, 6.EE.7



In 10–16, write and solve an equation to answer each question. Tell what the variable represents.

10. A wildlife reserve had 8 elephant calves born during the summer and now has 31 total elephants. How many elephants were in the reserve before summer began?

11. My sister is 14 years old. My brother says that his age minus 12 is equal to my sister's age. How old is my brother?

12. Kim bought a poster that cost $8.95 and some colored pencils. The total cost was $21.35. How much did the colored pencils cost?

13. Acme Cars sold 37 vehicles in June. How many compact cars were sold in June?

Acme Cars – June Sales	
Type of car	**Number sold**
SUV	8
Compact	?

14. Sandra wants to buy a new MP3 player that is on sale for $95. She has saved $73. How much more money does she need?

15. Ronald spent $123.45 on school clothes. He counted his money and discovered that he had $36.55 left. How much money did he originally have?

16. Brita withdrew $225 from her bank account. After her withdrawal, there was $548 left in her account. How much money did Brita have in her account before the withdrawal?

17. **Represent Real-World Problems** Write a real-world situation that can be represented by $15 + c = 17.50$. Tell what the variable represents. Then solve the equation and describe what your answer represents for the problem situation.

18. **Critique Reasoning** Paula solved the equation $7 + x = 10$ and got 17. She is not certain that her answer is correct. How could you explain her mistake to her?

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

19. **Multistep** A grocery store is having a sale this week. If you buy a 5-pound bag of apples for the regular price, you can get another bag for $1.49. If you buy a 5-pound bag of oranges at the regular price, you can get another bag for $2.49.

Product	Regular price
5-pound bag of apples	$2.99
5-pound bag of oranges	$3.99

a. Write an equation to find the discount for each situation using a for the amount of the discount for apples and r for the amount of the discount for oranges.

_____

b. Which fruit has a greater discount? Explain.

_____

_____

20. **Critical Thinking** An orchestra has twice as many woodwind instruments as brass instruments. There are a total of 150 brass and woodwind instruments.

a. Write two different addition equations that describe this situation. Use w for the number of woodwinds and b for the number of brass instruments.

_____

b. How many woodwinds and how many brass instruments must there be to satisfy the given information?

_____

21. **Look for a Pattern** Assume the following: $a + 1 = 2$, $b + 10 = 20$, $c + 100 = 200$, $d + 1{,}000 = 2{,}000$, ...

a. Solve each equation.

_____

b. What pattern do you notice in the values of the variables?

_____

_____

c. What would be the value of g if the pattern continued?

_____

LESSON 11.3

Multiplication and Division Equations

CA CC 6.EE.5

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. *Also 6.EE.6, 6.EE.7*

ESSENTIAL QUESTION

How do you solve equations that contain multiplication or division?

EXPLORE ACTIVITY Real World

CA CC 6.EE.6

Modeling Equations

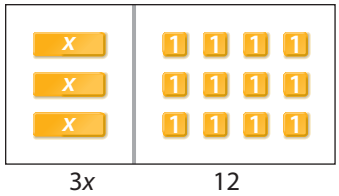
Deanna has a recipe for potato cakes that requires 12 eggs to make 3 batches of potato cakes. How many eggs are needed per batch?

Let x represent the number of eggs needed per batch.

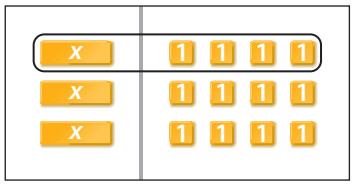
Number of batches	·	Number of eggs per batch	=	Total eggs
↓		↓		↓
3	·	x	=	12

To answer this question, you can use algebra tiles to solve $3x = 12$.

A Model $3x = 12$.



B There are 3 x tiles, so draw circles to separate the tiles into 3 equal groups. One group has been circled for you.



Math Talk
Mathematical Practices
Why is the solution to the equation the number of tiles in each group?

C How many 1 tiles are in each group? _____
This is the solution of the equation.

_____ eggs are needed per batch of potato cakes.

Reflect

1. **Look for a Pattern** Why does it make sense to arrange the twelve 1 tiles in 3 rows of 4 instead of any other arrangement of twelve 1 tiles, such as 2 rows of 6?

_____

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Using Division to Solve Equations

Separating the tiles on both sides of an equation mat into an equal number of groups models dividing both sides of an equation by the same number.

Division Property of Equality

You can divide both sides of an equation by the same nonzero number, and the two sides will remain equal.

When an equation contains multiplication, solve by dividing both sides of the equation by the same nonzero number.

My Notes

EXAMPLE 1

CA CC 6.EE.5

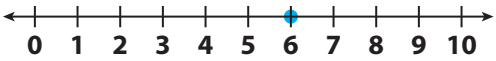
Solve each equation. Graph the solution on a number line.

A $9a = 54$

$9a = 54$ — Notice that 9 is multiplied by a.

$\frac{9a}{9} = \frac{54}{9}$ — Divide both sides of the equation by 9.

$a = 6$



Check: $9a = 54$

$9(6) \stackrel{?}{=} 54$ — Substitute 6 for a.

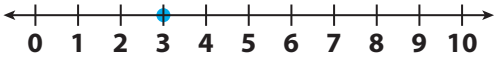
$54 \stackrel{?}{=} 54$ — Multiply on the left side.

B $18 = 6d$

$18 = 6d$ — Notice that 6 is multiplied by d.

$\frac{18}{6} = \frac{6d}{6}$ — Divide both sides of the equation by 6.

$3 = d$



Check: $18 = 6d$

$18 \stackrel{?}{=} 6(3)$ — Substitute 3 for d.

$18 \stackrel{?}{=} 18$ — Multiply on the right side.

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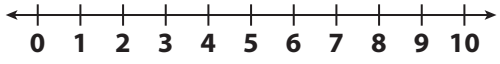
YOUR TURN

Solve the equation $3x = 21$. Graph the solution on a number line.

2. $x =$ _____



Using Multiplication to Solve Equations

When an equation contains division, solve by multiplying both sides of the equation by the same number.

Multiplication Property of Equality

You can multiply both sides of an equation by the same number, and the two sides will remain equal.

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EXAMPLE 2

CA CC 6.EE.5

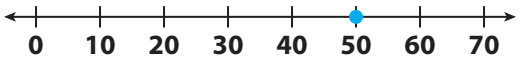
Solve each equation. Graph the solution on a number line.

A $\frac{x}{5} = 10$

$\frac{x}{5} = 10$ — Notice that x is divided by the number 5.

$5 \cdot \frac{x}{5} = 5 \cdot 10$ — Multiply both sides of the equation by 5.

$x = 50$



Check: $\frac{x}{5} = 10$

$\frac{50}{5} \stackrel{?}{=} 10$ — Substitute 50 for x.

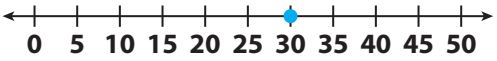
$10 \stackrel{?}{=} 10$ — Divide on the left side.

B $15 = \frac{r}{2}$

$15 = \frac{r}{2}$ — Notice that r is divided by the number 2.

$2 \cdot 15 = 2 \cdot \frac{r}{2}$ — Multiply both sides of the equation by 2.

$30 = r$



Check: $15 = \frac{r}{2}$

$15 \stackrel{?}{=} \frac{30}{2}$ — Substitute 30 for r.

$15 \stackrel{?}{=} 15$ — Divide on the right side.

Math Talk
Mathematical Practices

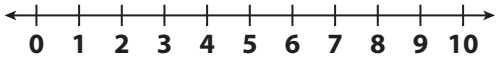
How is solving a multiplication equation similar to solving a division equation? How is it different?

YOUR TURN

Solve the equation $\frac{y}{9} = 1$. Graph the solution on a number line.

3. $\frac{y}{9} = 1$

$y =$ _____



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My Notes

Using Equations to Solve Problems

You can use equations to solve real-world problems.

EXAMPLE 3

Problem Solving

CA CC 6.EE.7

Juanita is scrapbooking. She usually completes about 9 pages per hour. One night last week she completed pages 23 through 47 in 2.5 hours. Did she work at her average rate?

Analyze Information

Identify the important information.

- Worked for 2.5 hours
- Starting page: 23 Ending page: 47
- Scrapbooking rate: 9 pages per hour

Formulate a Plan

- Solve an equation to find the number of pages Juanita can expect to complete.
- Compare the number of pages Juanita can expect to complete with the number of pages she actually completed.

Solve

Let n represent the number of pages Juanita can expect to complete in 2.5 hours if she works at her average rate of 9 pages per hour.

Write an equation.

$\frac{n}{2.5} = 9$ Write the equation.

$2.5 \cdot \frac{n}{2.5} = 2.5 \cdot 9$ Multiply both sides by 2.5.

$n = 22.5$

Juanita can expect to complete 22.5 pages in 2.5 hours.

Juanita completed pages 23 through 47, a total of 25 pages. Because $25 > 22.5$, she worked faster than her expected rate.

Justify and Evaluate

You used an equation to find the number of pages Juanita could expect to complete in 2.5 hours if she worked at her average rate. You found that she could complete 22.5 pages.

Since 22.5 pages is less than the 25 pages Juanita completed, she worked faster than her average rate.

The answer makes sense, because Juanita completed 25 pages in 2.5 hours, which is equivalent to a rate of 10 pages in 1 hour. Since $10 > 9$, you know that she worked faster than her average rate.

YOUR TURN

4. Tion divides his baseball cards equally among himself, his brother, and 3 friends. He has 9 cards left. How many does he give away? Write and solve an equation to solve the problem. Tell what the variable represents.

_____

_____

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Writing Real-World Problems

You can write a real-world problem for a given equation.

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EXAMPLE 4

Real World

CA CC 6.EE.7

Write a real-world problem for the equation $8x = 72$. Then solve the problem.

STEP 1 Examine each part of the equation.

x is the unknown value you want to find.

8 is multiplied by x.

$= 72$ means that after multiplying 8 and x, the result is 72.

STEP 2 Write a real-world situation that involves multiplying two quantities.

A hot air balloon flew at 8 miles per hour. Write and solve a multiplication equation to find the number of hours the balloon traveled if it covered a distance of 72 miles. Let x represent that number of hours.

STEP 3 Use the equation to solve the problem.

$8x = 72$

Divide both sides by 8.

$\frac{8x}{8} = \frac{72}{8}$

$x = 9$

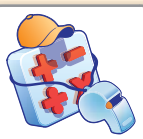
The balloon traveled for 9 hours.

YOUR TURN

5. Write a real-world problem for the equation $11x = 385$. Tell what the variable represents. Then solve the problem.

_____

_____



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Guided Practice

1. Caroline ran 15 miles in 5 days. She ran the same distance each day. Write and solve an equation to determine the number of miles she ran each day. (Explore Activity)

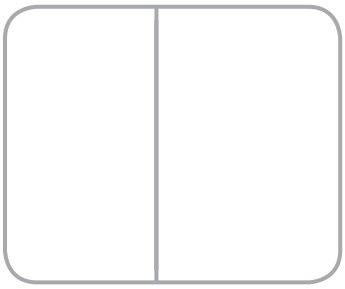
a. Let x represent the _____.

b.

Number of	·	Number of	=	
_____	·	_____	=	_____

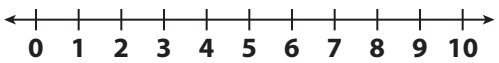
c. Draw algebra tiles to model the equation.

Caroline ran _____ miles each day.

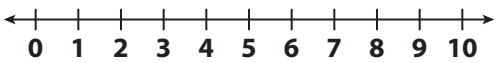


Solve each equation. Graph the solution on a number line. (Examples 1 and 2)

2. $x \div 3 = 3$; $x =$ _____



3. $4x = 32$; $x =$ _____



4. The area of the rectangle shown is 24 square inches. How much longer is its length than its width? (Example 3)

_____

_____

5. Write a real-world problem for the equation $15w = 45$. Tell what the variable represents. Then solve the problem. (Example 4)

_____

_____

_____

ESSENTIAL QUESTION CHECK-IN

6. How do you solve equations that contain multiplication or division?

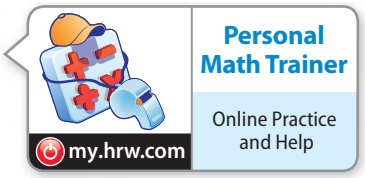
_____

_____

Name _____ Class _____ Date _____

11.3 Independent Practice

CA CC 6.EE.5, 6.EE.6, 6.EE.7



Solve each equation.

7. $0.7x = 42$ _____

8. $\frac{3}{4}x = \frac{1}{8}$ _____

9. $\frac{x}{4} = 2\frac{1}{4}$ _____

10. $\frac{x}{5} = 2.3$ _____

11. $x \div 3.2 = 4.5$ _____

In 12–17, write and solve an equation to answer each question. Tell what the variable represents.

12. Some friends made a salad for a party. There were 28 people at the party including the friends. They bought enough cherry tomatoes for everyone to get 3 cherry tomatoes each. How many cherry tomatoes did the friends buy?

_____

_____

13. Carmen participated in a read-a-thon. Mr. Cole pledged $4.00 per book and gave Carmen $44. How many books did Carmen read?

_____

14. Lee drove 420 miles and used 15 gallons of gasoline. How many miles did Lee's car travel per gallon of gasoline?

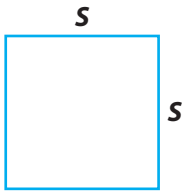
_____

_____

15. On some days, Melvin commutes 3.5 hours per day to the city for business meetings. Last week he commuted for a total of 14 hours. How many days did he commute to the city?

_____

16. Dharmesh has a square garden with a perimeter of 132 feet. Is the area of the garden greater than 1,000 square feet?



_____

_____

_____

_____

_____

17. Ingrid walked her dog and washed her car. She spent 3 times as much time washing her car as she did walking her dog. It took Ingrid $1\frac{1}{2}$ hours to wash her car. How long did it take Ingrid to walk her dog?

_____

18. **Representing Real-World Problems** Write and solve a problem involving money that can be solved with a multiplication equation. Tell what the variable represents.

_____

_____

_____

_____

_____

_____

_____

19. **Representing Real-World Problems** Write and solve a problem involving money that can be solved with a division equation and has a solution of 1,350.

H.O.T. FOCUS ON HIGHER ORDER THINKING

20. **Communicating Mathematical Ideas** Explain why $7 \cdot \frac{x}{7} = x$. How does your answer help you solve a division equation such as $\frac{x}{7} = 2$?

21. **Critical Thinking** A number tripled and tripled again is 729. What is the number? Show your work.

22. **Multistep** Andre has 4 times as many model cars as Peter, and Peter has one-third as many model cars as Jade. Andre has 36 model cars.

a. Write and solve an equation to find how many model cars Peter has.

b. Using your answer from part **a**, write and solve an equation to find how many model cars Jade has.

23. **Persevere in Problem Solving** The area of a rectangle is 42 square inches and one side is 12 inches long. Find the perimeter of the rectangle. Show your work.

Work Area

LESSON

11.4 Writing Inequalities

CA CC 6.EE.8

Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. *Also 6.EE.5, 6.EE.6*

ESSENTIAL QUESTION

How can you use inequalities to represent real-world constraints or conditions?

EXPLORE ACTIVITY Real World

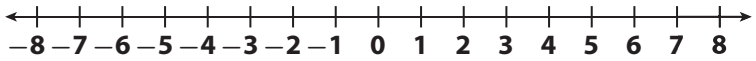
CA CC 6.EE.8, 6.EE.5

Using Inequalities to Describe Quantities

An **inequality** is a mathematical sentence that shows the relationship between quantities that are not equal. You can use the symbols in the table to write inequalities that describe quantities with many values.

Symbol	Meaning	Word Phrases
$<$	Is less than	Fewer than, below
$>$	Is greater than	More than, above
$\leq$	Is less than or equal to	At most, no more than
$\geq$	Is greater than or equal to	At least, no less than

A The lowest temperature ever recorded in Florida was $-2\,^{\circ}\text{F}$. Graph this temperature on the number line.



B The temperatures $0\,^{\circ}\text{F}$, $3\,^{\circ}\text{F}$, $6\,^{\circ}\text{F}$, $5\,^{\circ}\text{F}$, and $-1\,^{\circ}\text{F}$ have also been recorded in Florida. Graph these temperatures on the number line.

C How do the temperatures in **B** compare with -2? How can you see this relationship on the number line?

_____

D How many other numbers have the same relationship to -2 as the temperatures in **B**? Give some examples.

_____

E Suppose you could graph all of the possible answers to **D** on a number line. What would the graph look like?

_____

F Let x represent all the possible answers to **D**.

Complete this inequality: x ☐ -2

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Graphing the Solutions of an Inequality

A **solution of an inequality** that contains a variable is any value of the variable that makes the inequality true. For example, 7 is a solution of $x > -2$, since $7 > -2$ is a true statement.

EXAMPLE 1

CA CC 6.EE.5

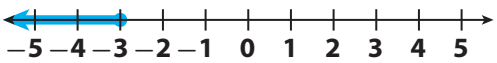
Graph the solutions of each inequality. Check the solutions.

A $y \leq -3$

STEP 1 Draw a solid circle at -3 to show that -3 is a solution.

STEP 2 Shade the number line to the left of -3 to show that numbers less than -3 are solutions.

Use a solid circle for an inequality that uses $\geq$ or $\leq$.



STEP 3 Check your solution.

Choose a number that is on the shaded section of the number line, such as -4. Substitute -4 for y.

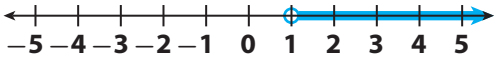
$-4 \leq -3$ -4 is less than -3, so -4 is a solution.

B $1 < m$

STEP 1 Draw an empty circle at 1 to show that 1 is not a solution.

STEP 2 Shade the number line to the right of 1 to show that numbers greater than 1 are solutions.

Use an open circle for an inequality that uses $>$ or $<$.



STEP 3 Check your answer.
Substitute 2 for m.
$1 < 2$ 1 is less than 2, so 2 is a solution.

Math Talk
Mathematical Practices
Is $-4\frac{1}{4}$ a solution of $y \leq -3$? Is -5.6?

My Notes

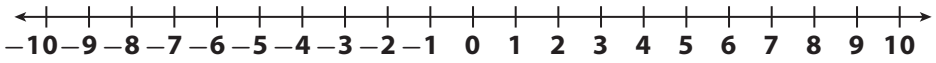
Reflect

1. **Critique Reasoning** Inez says you can rewrite $1 < m$ as $m > 1$. Do you agree? Explain.

2. **Analyze Relationships** How is $x < 5$ different from $x \leq 5$?

YOUR TURN

3. Graph the solutions of the inequality $t \leq -4$.



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Writing Inequalities

You can write an inequality to model the relationship between an algebraic expression and a number, or to represent certain real-world situations. The variable in an inequality can represent an infinite number of numbers.

EXAMPLE 2

Real World

CA CC 6.EE.8, 6.EE.6

A Write an inequality that represents the phrase *the sum of* y *and 2 is greater than 5*. Draw a graph to represent the inequality.

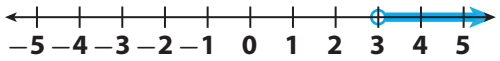
STEP 1 Write the inequality.

The sum of y and 2 is greater than 5.

$y + 2 > 5$

STEP 2 Graph the solution.

Use mental math: For $y + 2$ to have a value greater than 5, y must be a number greater than 3.



y is any number that meets the condition greater than 3.

STEP 3 Check your solution by substituting a number greater than 3, such as 4, into the original inequality.

$4 + 2 > 5$ Substitute 4 for y.

$6 > 5$ 6 is greater than 5, so 4 is a solution.

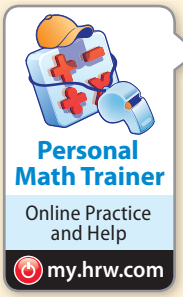
Math Talk
Mathematical Practices
Describe the numbers that the variable in **A** represents.

B To test the temperature rating of a coat, a scientist keeps the temperature below 5 °C. Write and graph an inequality to represent this situation.

STEP 1 Write the inequality. Let t represent the temperature in the lab.

$t < 5$ The temperature must be less than 5 °C.

STEP 2 Graph the inequality.



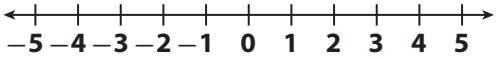
YOUR TURN

4. Write an inequality that represents the phrase *the sum of 1 and* y *is greater than or equal to 3* . Check to see if $y = 1$ is a solution.

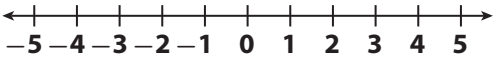
5. The temperature in February was at most 6 °F. Write and graph an inequality to represent the situation. Tell what the variable represents.

Guided Practice

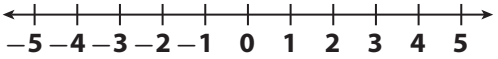
1. Graph $1 \leq x$. Use the graph to determine which of these numbers are solutions of the inequality: -1, 3, 0, 1 (Explore Activity and Example 1)



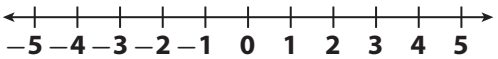
2. Graph $-3 > z$. Check the graph using substitution. (Example 1)



3. Write an inequality that represents the phrase *the product of 3 and* x *is greater than 6*. Draw a graph that represents the inequality, and check your solution. (Example 2)



4. During hibernation, a garter snake's body temperature never goes below 3 °C. Write and graph an inequality that represents this situation. Tell what the variable represents. (Example 2)



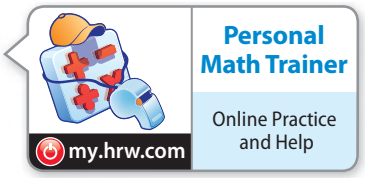
ESSENTIAL QUESTION CHECK-IN

5. Write an inequality to represent this situation: Nina wants to take at least $15 to the movies. How did you decide which inequality symbol to use?

Name _____ Class _____ Date _____

11.4 Independent Practice

CA CC 6.EE.5, 6.EE.6, 6.EE.8

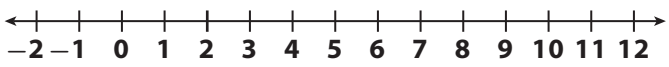


6. Which of the following numbers are solutions of $x \geq 0$?

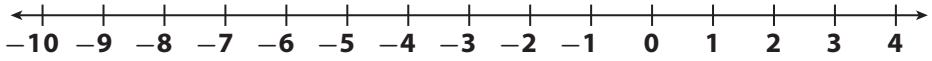
$-5, 0.03, -1, 0, 1.5, -6, \frac{1}{2}$ _____

Graph the solutions of each inequality. Check the solutions.

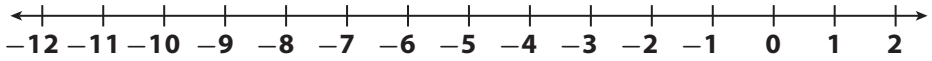
7. $t - 8 \leq 0$



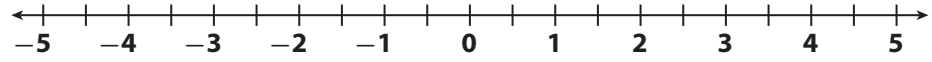
8. $-7 < h$



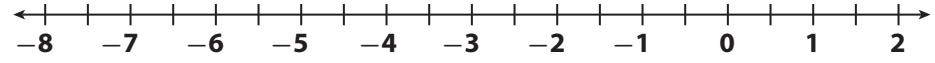
9. $x \geq -9$



10. $2n > 5$

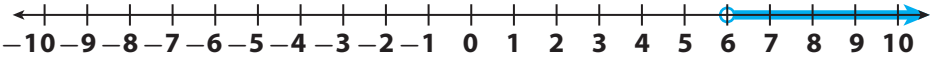


11. $-4\frac{1}{2} > x$

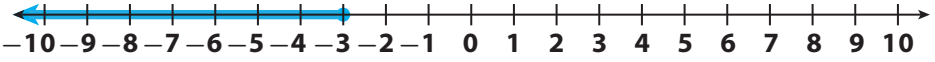


Write an inequality that matches the number line model.

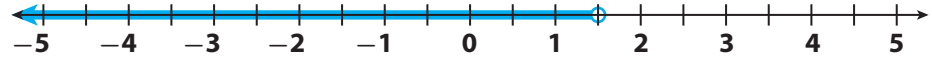
12. _____



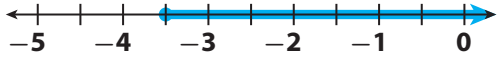
13. _____



14. _____

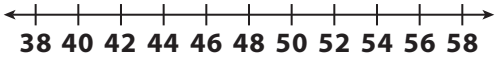


15. _____



16. A child must be at least 48 inches tall to ride a roller coaster.

a. Write and graph an inequality to represent this situation. Tell what the variable represents.



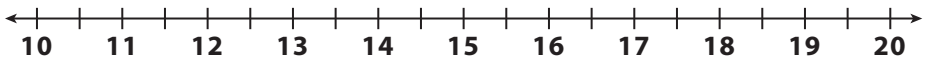
_____

b. Can a child who is 46 inches tall ride the roller coaster? Explain.

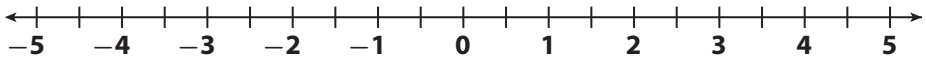
_____

Write and graph an inequality to represent each situation. Tell what the variable represents.

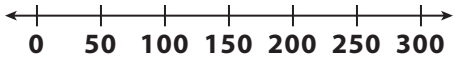
17. The stock is worth at least $14.50. _____



18. The temperature is less than 3.5 °F. _____



19. The goal of the fundraiser is to make more than $150. _____



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H.O.T. FOCUS ON HIGHER ORDER THINKING

20. Communicate Mathematical Ideas Explain how to graph the inequality $8 \geq y$.

_____

_____

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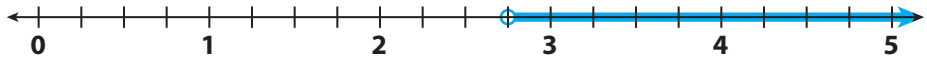
21. Represent Real-World Problems The number line shows an inequality. Describe a real-world situation that the inequality could represent.



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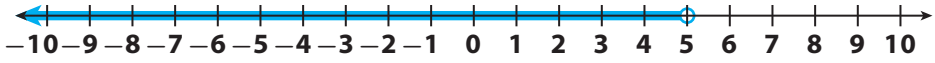
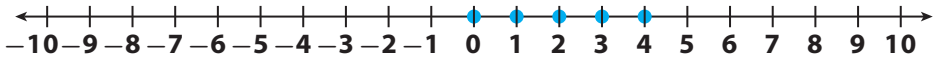
22. Critique Reasoning Natasha is trying to represent the following situation with a number line model: There are fewer than 5 students in the cafeteria. She has come up with two possible representations, shown below. Which representation is correct? Why?





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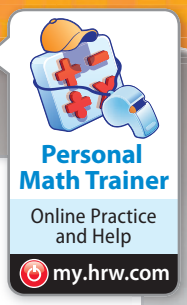
Work Area

MODULE QUIZ

Ready to Go On?



11.1 Writing Equations to Represent Situations

Determine whether the given value is a solution of the equation.

1. $\frac{b}{12} = 5; b = 60$ _____

2. $7w = 87; w = 12$ _____

Write an equation to represent the situation.

3. The number e of eggs in the refrigerator decreased by 5 equals 18.

_____

11.2 Addition and Subtraction Equations

Solve each equation.

4. $r - 38 = 9$ _____

5. $h + \frac{1}{2} = 3\frac{1}{4}$ _____

6. $n + 75 = 155$ _____

7. $q - 17 = 18$ _____

11.3 Multiplication and Division Equations

Solve each equation.

8. $8z = 11.2$ _____

9. $\frac{d}{14} = 7$ _____

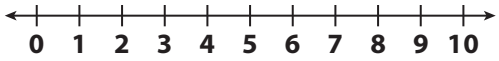
10. $\frac{f}{28} = 24$ _____

11. $3a = 57$ _____

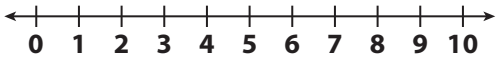
11.4 Writing Inequalities

Write an inequality to represent each situation, then graph the solutions.

12. There are fewer than 8 gallons of gas in the tank. _____



13. There are at least 3 slices of bread left in the bag. _____



ESSENTIAL QUESTION

14. How can you solve problems involving equations that contain addition, subtraction, multiplication, or division?

_____

_____

MODULE 11
MIXED REVIEW

Assessment Readiness

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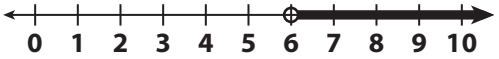
1. Consider each inequality and number line representation.

Select Yes or No in A–C to tell if the inequality is represented correctly on the number line.

A. $x < 6$

◯ Yes ◯ No

B. $x \geq 6$



◯ Yes ◯ No

C. $x \leq 6$

◯ Yes ◯ No

2. Choose True or False for A–C.

A. $y = 7$ is the solution of $y + 7 = 0$. ◯ True ◯ False

B. $y = 21$ is the solution of $\frac{y}{3} = 7$. ◯ True ◯ False

C. $y = 5$ is the solution of $5y = 1$. ◯ True ◯ False

3. Katie has read 34% of a book. If she has read 85 pages, does she have more than or fewer than 125 pages left to read? Explain.

4. Sylvia can work at most 15 hours per week at her part-time job. Last week she worked 12 hours and earned $84. She hopes to earn more than $100 next week. Explain how to use equations and inequalities to determine whether she can do so.