Solving Systems of Linear Equations

MODULE 8

CALIFORNIA

ESSENTIAL QUESTION

How can you use systems of equations to solve real-world problems?

Real-World Video

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The distance contestants in a race travel over time can be modeled by a system of equations. Solving such a system can tell you when one contestant will overtake another who has a head start, as in a boating race or marathon.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Simplify Algebraic Expressions

EXAMPLE Simplify $5 - 4y + 2x - 6 + y$.

$-4y + y + 2x - 6 + 5$ *Group like terms.*

$-3y + 2x - 1$ *Combine like terms.*

Simplify.

1. $14x - 4x + 21$ __________

2. $-y - 4x + 4y$ __________

3. $5.5a - 1 + 21b + 3a$ __________

4. $2y - 3x + 6x - y$ __________

Graph Linear Equations

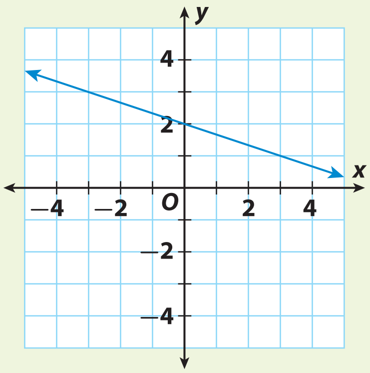
EXAMPLE Graph $y = -\frac{1}{3}x + 2$.

Step 1: Make a table of values.

x	$y = -\frac{1}{3}x + 2$	(x, y)
0	$y = -\frac{1}{3}(0) + 2 = 2$	(0, 2)
3	$y = -\frac{1}{3}(3) + 2 = 1$	(3, 1)

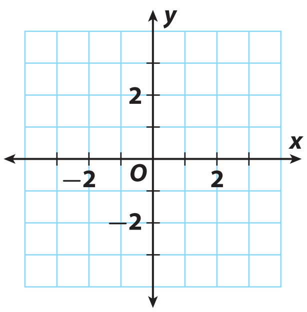
Step 2: Plot the points.

Step 3: Connect the points with a line.

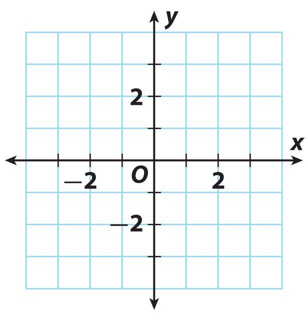


Graph each equation.

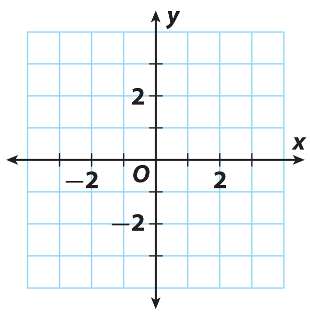
5. $y = 4x - 1$



6. $y = \frac{1}{2}x + 1$



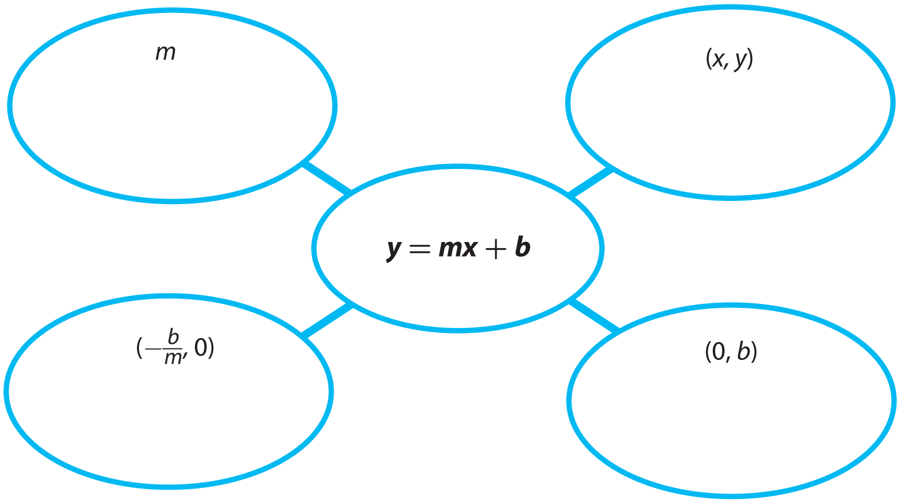
7. $y = -x$



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.



Vocabulary

Review Words

linear equation *(ecuación lineal)*
✔ ordered pair *(par ordenado)*
✔ slope *(pendiente)*
slope-intercept form *(forma pendiente intersección)*
x-axis *(eje x)*
✔ *x*-intercept *(intersección con el eje x)*
y-axis *(eje y)*
✔ *y*-intercept *(intersección con el eje y)*

Preview Words

solution of a system of equations *(solución de un sistema de ecuaciones)*
system of equations *(sistema de ecuaciones)*

Understand Vocabulary

Complete the sentences using the preview words.

1. A ______________________________ is any ordered pair that satisfies all the equations in a system.

2. A set of two or more equations that contain two or more variables is called a ______________________________.

Active Reading

Four-Corner Fold Before beginning the module, create a four-corner fold to help you organize what you learn about solving systems of equations. Use the categories "Solving by Graphing," "Solving by Substitution," "Solving by Elimination," and "Solving by Multiplication." As you study this module, note similarities and differences among the four methods. You can use your four-corner fold later to study for tests and complete assignments.

GETTING READY FOR

Solving Systems of Linear Equations

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

CA CC 8.EE.8a

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Key Vocabulary

solution of a system of equations *(solución de un sistema de ecuaciones)* A set of values that make all equations in a system true.

system of equations *(sistema de ecuaciones)* A set of two or more equations that contain two or more variables.

What It Means to You

You will understand that the points of intersection of two or more graphs represent the solution to a system of linear equations.

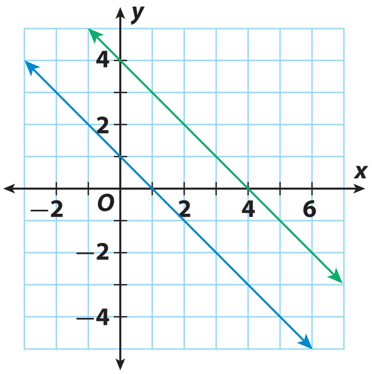
EXAMPLE 8.EE.8a, 8.EE.8b

Use the elimination method.

A.

$$\begin{aligned} -x &= -1 + y \\ \underline{x + y} &\underline{= 4} \\ y &= y + 3 \end{aligned}$$

This is never true, so the system has no solution. The graphs never intersect.



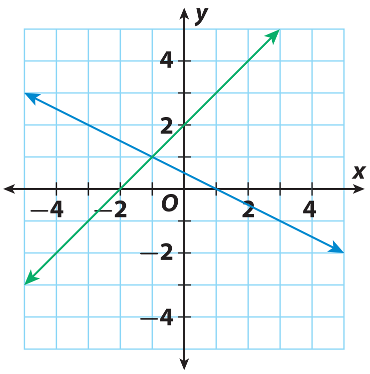
Use the substitution method.

B.

$$2y + x = 1$$
$$y - 2 = x$$

$$2y + (y - 2) = 1$$
$$3y - 2 = 1$$
$$y = 1$$

$$x = y - 2$$
$$x = 1 - 2$$
$$x = -1$$



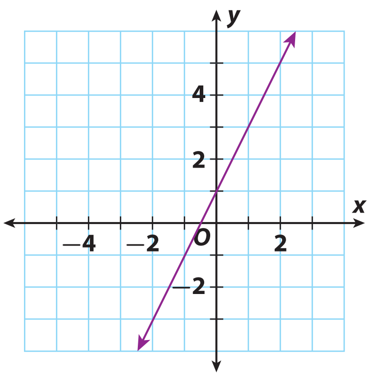
Only one solution: $x = -1$, $y = 1$.
The graphs intersect at the point $(-1, 1)$.

Use the multiplication method.

C.

$$3y - 6x = 3$$
$$y - 2x = 1$$

$$3y - 6x = 3$$
$$\underline{3y - 6x = 3}$$
$$0 = 0$$



This is always true. So the system has infinitely many solutions. The graphs are the same line.

LESSON **8.1**

Solving Systems of Linear Equations by Graphing

CA CC 8.EE.8a

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. *Also 8.EE.8, 8.EE.8c*

ESSENTIAL QUESTION

How can you solve a system of equations by graphing?

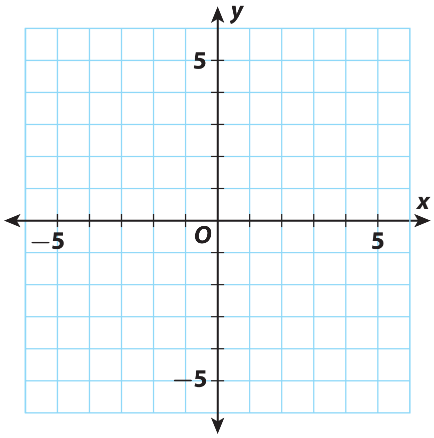
EXPLORE ACTIVITY CA CC 8.EE.8a

Investigating Systems of Equations

You have learned several ways to graph a linear equation in slope-intercept form. For example, you can use the slope and y-intercept or you can find two points that satisfy the equation and connect them with a line.

Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

A Graph the pair of equations together: $\begin{cases} y = 3x - 2 \\ y = -2x + 3 \end{cases}$.



B Explain how to tell whether $(2, -1)$ is a solution of the equation $y = 3x - 2$ without using the graph.

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C Explain how to tell whether $(2, -1)$ is a solution of the equation $y = -2x + 3$ without using the graph.

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D Use the graph to explain whether $(2, -1)$ is a solution of each equation.

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E Determine if the point of intersection is a solution of both equations.

Point of intersection: $(\square, \square)$

$y = 3x - 2$

$\square = 3\square - 2$

$1 = \square$

$y = -2x + 3$

$\square = -2\square + 3$

$1 = \square$

The point of intersection **is / is not** the solution of both equations.

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My Notes

Solving Systems Graphically

An ordered pair (x, y) is a solution of an equation in two variables if substituting the x- and y-values into the equation results in a true statement. A **system of equations** is a set of equations that have the same variables. An ordered pair is a **solution of a system of equations** if it is a solution of every equation in the set.

Since the graph of an equation represents all ordered pairs that are solutions of the equation, if a point lies on the graphs of two equations, the point is a solution of both equations and is, therefore, a solution of the system.

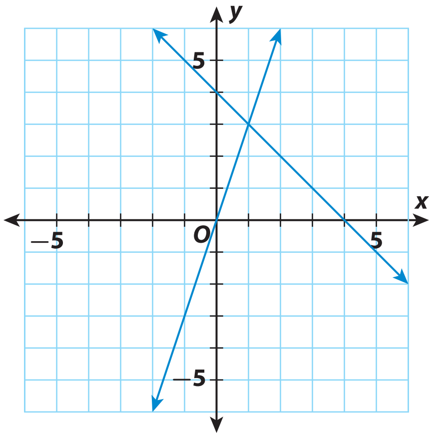
EXAMPLE 1

CA CC 8.EE.8

Solve each system by graphing.

A $\begin{cases} y = -x + 4 \\ y = 3x \end{cases}$

STEP 1 Start by graphing each equation.



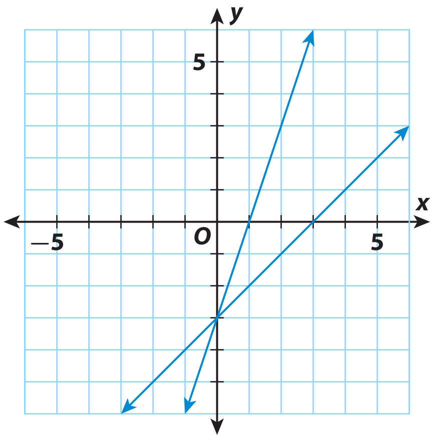
STEP 2 Find the point of intersection of the two lines. It appears to be (1, 3). Substitute to check if it is a solution of both equations.

$y = -x + 4$	$y = 3x$
$3 \stackrel{?}{=} -(1) + 4$	$3 \stackrel{?}{=} 3(1)$
$3 = 3$ ✓	$3 = 3$ ✓

The solution of the system is (1, 3).

B $\begin{cases} y = 3x - 3 \\ y = x - 3 \end{cases}$

STEP 1 Start by graphing each equation.



STEP 2 Find the point of intersection of the two lines. It appears to be (0, –3). Substitute to check if it is a solution of both equations.

$y = 3x - 3$	$y = x - 3$
$-3 \stackrel{?}{=} 3(0) - 3$	$-3 \stackrel{?}{=} 0 - 3$
$-3 = -3$ ✓	$-3 = -3$ ✓

The solution of the system is (0, –3).

Reflect

1. **What If?** If you want to include another linear equation in the system of equations in part A of Example 1 without changing the solution, what must be true about the graph of the equation?

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2. **Analyze Relationships** Suppose you include the equation $y = -2x - 3$ in the system of equations in part B of Example 1. What effect will this have on the solution of the system? Explain your reasoning.

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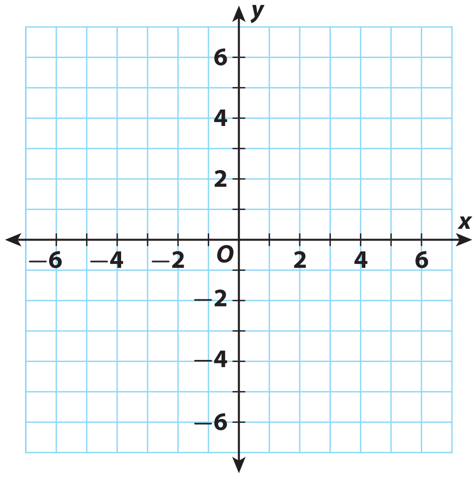
YOUR TURN

Solve each system by graphing. Check by substitution.

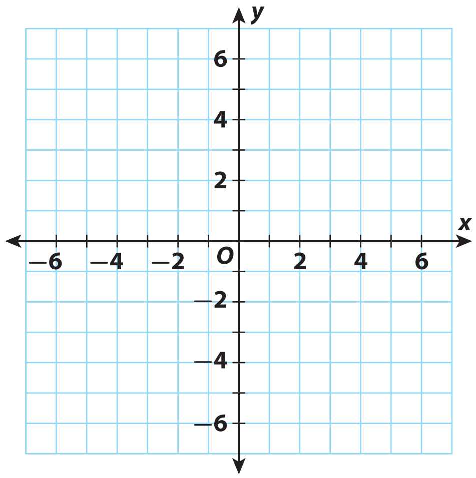
3. $\begin{cases} y = -x + 2 \\ y = -4x - 1 \end{cases}$ ____________

Check:



4. $\begin{cases} y = -2x + 5 \\ y = 3x \end{cases}$ ____________

Check:



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Solving Problems Using Systems of Equations

When using graphs to solve a system of equations, it is best to rewrite both equations in slope-intercept form for ease of graphing.

To write an equation in slope-intercept form starting from $ax + by = c$:

$$ax + by = c$$

$by = c - ax$ Subtract ax from both sides.

$y = \frac{c}{b} - \frac{ax}{b}$ Divide both sides by b.

$y = -\frac{a}{b}x + \frac{c}{b}$ Rearrange the equation.

EXAMPLE 2 Real World

CA CC 8.EE.8c, 8.EE.8

Keisha and her friends visit the concession stand at a football game. The stand charges $2 for a sandwich and $1 for a lemonade. The friends buy a total of 8 items for $11. Tell how many sandwiches and how many lemonades they bought.

STEP 1 Let x represent the number of sandwiches they bought and let y represent the number of lemonades they bought.

Write an equation representing the **number of items they purchased.**

Number of sandwiches	+	Number of lemonades	=	Total items
x	+	y	=	8

Write an equation representing the **money spent on the items.**

Cost of 1 sandwich times number of sandwiches	+	Cost of 1 lemonade times number of lemonades	=	Total cost
$2x$	+	$1y$	=	11

STEP 2 Write the equations in slope-intercept form. Then graph.

$x + y = 8$

$y = 8 - x$

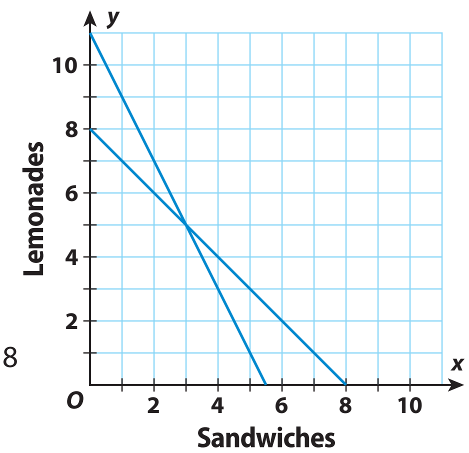
$y = -x + 8$

$2x + 1y = 11$

$1y = 11 - 2x$

$y = -2x + 11$

Graph the equations $y = -x + 8$ and $y = -2x + 11$.



STEP 3 Use the graph to identify the solution of the system of equations. Check your answer by substituting the ordered pair into both equations.

Apparent solution: (3, 5)

Check:

$$x + y = 8 \qquad\qquad 2x + y = 11$$
$$3 + 5 \stackrel{?}{=} 8 \qquad\qquad 2(3) + 5 \stackrel{?}{=} 11$$
$$8 = 8 \checkmark \qquad\qquad 11 = 11 \checkmark$$

The point (3, 5) is a solution of both equations.

STEP 4 Interpret the solution in the original context.

Keisha and her friends bought 3 sandwiches and 5 lemonades.

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Reflect

5. **Conjecture** Why do you think the graph is limited to the first quadrant?

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YOUR TURN

6. During school vacation, Marquis wants to go bowling and to play laser tag. He wants to play 6 total games but needs to figure out how many of each he can play if he spends exactly \$20. Each game of bowling is \$2 and each game of laser tag is \$4.

a. Let x represent the number of games Marquis bowls and let y represent the number of games of laser tag Marquis plays. Write a system of equations that describes the situation. Then write the equations in slope-intercept form.

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b. Graph the solutions of both equations.

c. How many games of bowling and how many games of laser tag will Marquis play?

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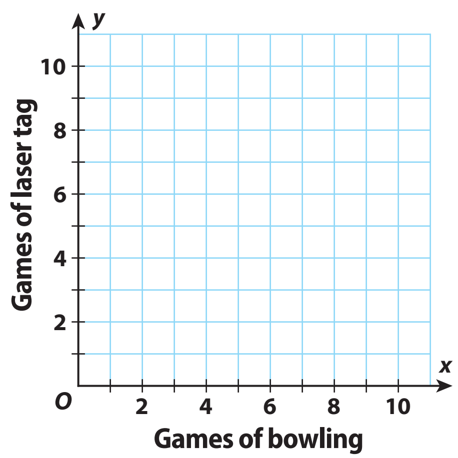

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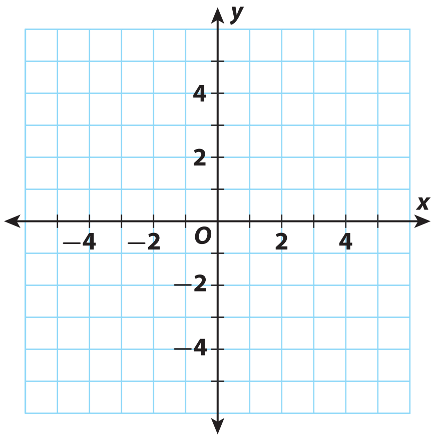
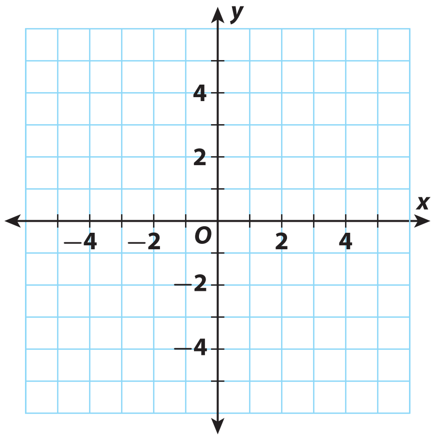
Guided Practice

Solve each system by graphing. (Examples 1 and 2)

1. $\begin{cases} y = 3x - 4 \\ y = x + 2 \end{cases}$ ________________

2. $\begin{cases} 2x + y = 4 \\ -3x + 9y = -6 \end{cases}$ ________________





3. **Mrs. Morales wrote a test with 15 questions covering spelling and vocabulary. Spelling questions (x) are worth 5 points and vocabulary questions (y) are worth 10 points. The maximum number of points possible on the test is 100.** (Example 2)

a. Write an equation in slope-intercept form to represent the number of questions on the test.

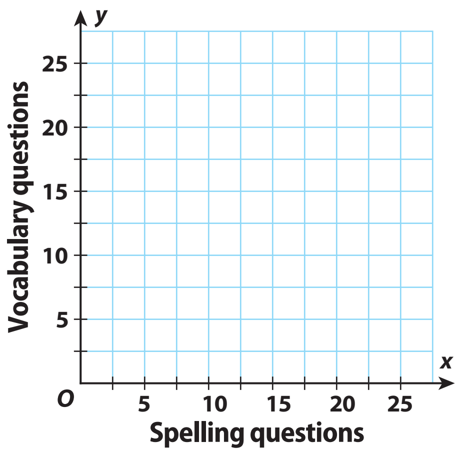
____

b. Write an equation in slope-intercept form to represent the total number of points on the test.

____

c. Graph the solutions of both equations.

d. Use your graph to tell how many of each question type are on the test.



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ESSENTIAL QUESTION CHECK-IN

4. When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

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Name ______________________ Class ______________ Date ______________

8.1 Independent Practice

CA CC 8.EE.8, 8.EE.8a, 8.EE.8c

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5. **Vocabulary** A ______________ is a set of equations that have the same variables.

6. Eight friends started a business. They will wear either a baseball cap or a shirt imprinted with their logo while working. They want to spend exactly $36 on the shirts and caps. Shirts cost $6 each and caps cost $3 each.

a. Write a system of equations to describe the situation. Let *x* represent the number of shirts and let *y* represent the number of caps.

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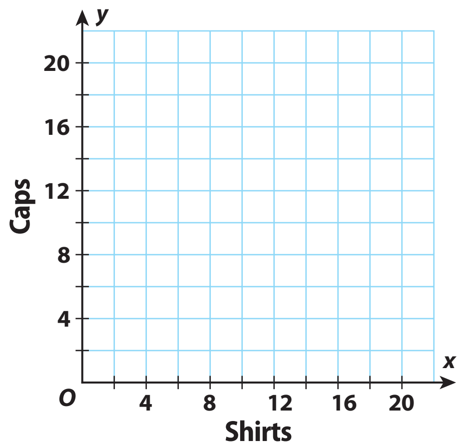
b. Graph the system. What is the solution and what does it represent?

Business Logo Wear



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7. **Multistep** The table shows the cost for bowling at two bowling alleys.

	Shoe Rental Fee	Cost per Game
Bowl-o-Rama	$2.00	$2.50
Bowling Pinz	$4.00	$2.00

a. Write a system of equations, with one equation describing the cost to bowl at Bowl-o-Rama and the other describing the cost to bowl at Bowling Pinz. For each equation, let *x* represent the number of games played and let *y* represent the total cost.

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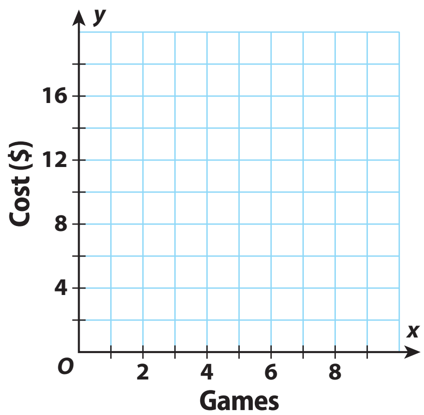
b. Graph the system. What is the solution and what does it represent?

Cost of Bowling



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8. **Multi-Step** Jeremy runs 7 miles per week and increases his distance by 1 mile each week. Tony runs 3 miles per week and increases his distance by 2 miles each week. In how many weeks will Jeremy and Tony be running the same distance? What will that distance be?

____

9. **Critical Thinking** Write a real-world situation that could be represented by the system of equations shown below.

$$\begin{cases} y = 4x + 10 \\ y = 3x + 15 \end{cases}$$

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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

10. **Multistep** The table shows two options provided by a high-speed Internet provider.

	Setup Fee ($)	Cost per Month ($)
Option 1	50	30
Option 2	No setup fee	$40

a. In how many months will the total cost of both options be the same? What will that cost be?

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b. If you plan to cancel your Internet service after 9 months, which is the cheaper option? Explain.

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11. **Draw Conclusions** How many solutions does the system formed by $x - y = 3$ and $ay - ax + 3a = 0$ have for a nonzero number a? Explain.

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LESSON 8.2

Solving Systems by Substitution

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION

How do you use substitution to solve a system of linear equations?

Solving a Linear System by Substitution

The **substitution method** is used to solve systems of linear equations by solving an equation for one variable and then substituting the resulting expression for that variable into the other equation. The steps for this method are as follows:

1. Solve one of the equations for one of its variables.
2. Substitute the expression from step 1 into the other equation and solve for the other variable.
3. Substitute the value from step 2 into either original equation and solve to find the value of the variable in step 1.

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EXAMPLE 1

CA CC 8.EE.8b

Solve the system of linear equations by substitution. Check your answer.

$$\begin{cases} -3x + y = 1 \\ 4x + y = 8 \end{cases}$$

STEP 1 Solve an equation for one variable.

$-3x + y = 1$ *Select one of the equations.*

$y = 3x + 1$ *Solve for the variable y. Isolate y on one side.*

STEP 2 Substitute the expression for y in the other equation and solve.

$4x + (3x + 1) = 8$ *Substitute the expression for the variable y.*

$7x + 1 = 8$ *Combine like terms.*

$7x = 7$ *Subtract 1 from each side.*

$x = 1$ *Divide each side by 7.*

STEP 3 Substitute the value of x you found into one of the equations and solve for the other variable, y.

$-3(1) + y = 1$ *Substitute the value of x into the first equation.*

$-3 + y = 1$ *Simplify.*

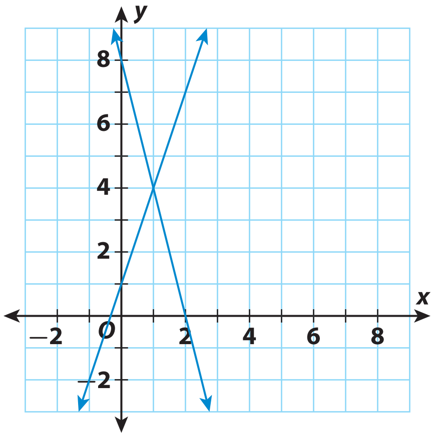
$y = 4$ *Add 3 to each side.*

So, (1, 4) is the solution of the system.

My Notes

STEP 4 Check the solution by graphing.

$-3x + y = 1$ | $4x + y = 8$
x-intercept: $-\frac{1}{3}$ | x-intercept: 2
y-intercept: 1 | y-intercept: 8



The point of intersection is (1, 4).

My Notes

Reflect

1. **Justify Reasoning** Is it more efficient to solve $-3x + y = 1$ for x? Why or why not?

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2. Is there another way to solve the system?

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3. What is another way to check your solution?

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YOUR TURN

Solve each system of linear equations by substitution.

4. $\begin{cases} 3x + y = 11 \\ -2x + y = 1 \end{cases}$ ______

5. $\begin{cases} 2x - 3y = -24 \\ x + 6y = 18 \end{cases}$ ______

6. $\begin{cases} x - 2y = 5 \\ 3x - 5y = 8 \end{cases}$ ______

Using a Graph to Estimate the Solution of a System

You can use a graph to estimate the solution of a system of equations before solving the system algebraically.

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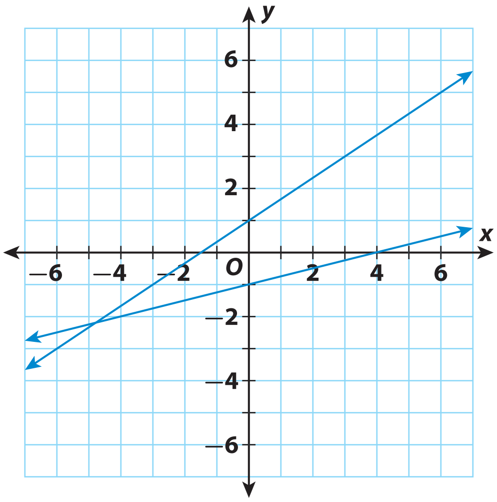
EXAMPLE 2

CA CC 8.EE.8b

Solve the system $\begin{cases} x - 4y = 4 \\ 2x - 3y = -3 \end{cases}$.

STEP 1 Sketch a graph of each equation by substituting values for x and generating values of y.



STEP 2 Find the intersection of the lines. The lines appear to intersect near $(-5, -2)$.

Math Talk Mathematical Practices

In Step 2, how can you tell that $(-5, -2)$ is not the solution?

STEP 3 Solve the system algebraically.

Solve $x - 4y = 4$ for x.

$$x - 4y = 4$$
$$x = 4 + 4y$$

Substitute to find y.

$$2(4 + 4y) - 3y = -3$$
$$8 + 8y - 3y = -3$$
$$8 + 5y = -3$$
$$5y = -11$$
$$y = -\frac{11}{5}$$

Substitute to find x.

$$x = 4 + 4y$$
$$= 4 + 4\left(-\frac{11}{5}\right)$$
$$= \frac{20 - 44}{5}$$
$$= -\frac{24}{5}$$

The solution is $\left(-\frac{24}{5}, -\frac{11}{5}\right)$.

STEP 4 Use the estimate you made using the graph to judge the reasonableness of your solution.

$-\frac{24}{5}$ is close to the estimate of -5, and $-\frac{11}{5}$ is close to the estimate of -2, so the solution seems reasonable.

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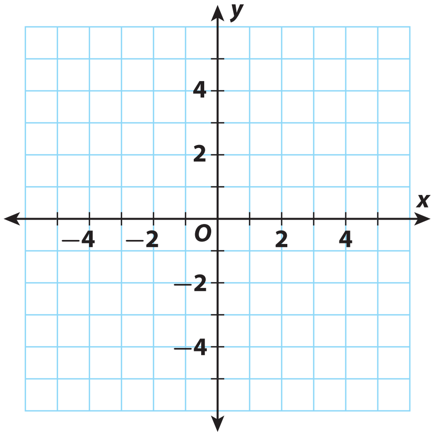
YOUR TURN

7. Estimate the solution of the system $\begin{cases} x + y = 4 \\ 2x - y = 6 \end{cases}$ by sketching a graph of each linear function. Then solve the system algebraically. Use your estimate to judge the reasonableness of your solution.



The estimated solution is ______________.

The algebraic solution is ______________.

The solution **is/is not** reasonable because

____

____

Solving Problems with Systems of Equations

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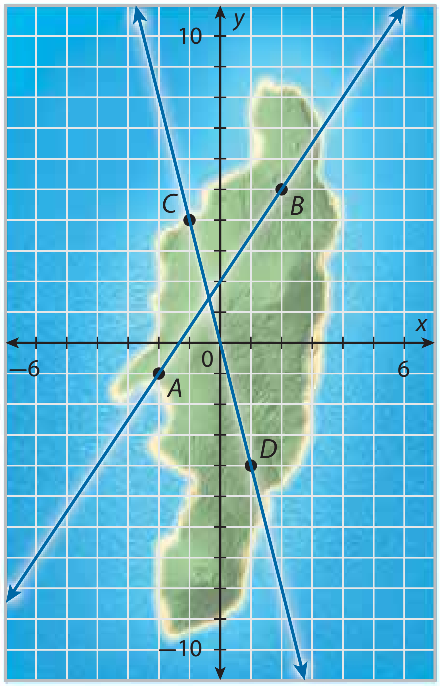
EXAMPLE 3 Real World

CA CC 8.EE.8c

As part of Class Day, the eighth grade is doing a treasure hunt. Each team is given the following riddle and map. At what point is the treasure located?

There's pirate treasure to be found. So search on the island, all around. Draw a line through *A* and *B*. Then another through *C* and *D*. Dance a jig, "X" marks the spot. Where the lines intersect, that's the treasure's plot!



STEP 1 Give the coordinates of each point and find the slope of the line through each pair of points.

A: (−2, −1) C: (−1, 4)

B: (2, 5) D: (1, −4)

Slope:

$$\frac{5-(-1)}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$$

Slope:

$$\frac{-4-4}{1-(-1)} = \frac{-8}{2} = -4$$

Math Talk
Mathematical Practices

Where do the lines appear to intersect? How is this related to the solution?

STEP 2 Write equations in slope-intercept form describing the line through points A and B and the line through points C and D.

Line through A and B:

Use the slope and a point to find b.

$$5 = \left(\frac{3}{2}\right)2 + b$$
$$b = 2$$

The equation is $y = \frac{3}{2}x + 2$.

Line through C and D:

Use the slope and a point to find b.

$$4 = -4(-1) + b$$
$$b = 0$$

The equation is $y = -4x$.

STEP 3 Solve the system algebraically.

Substitute $\frac{3}{2}x + 2$ for y in $y = -4x$ to find x.

$$\frac{3}{2}x + 2 = -4x$$
$$\frac{11}{2}x = -2$$
$$x = -\frac{4}{11}$$

Substitute to find x.

$y = -4\left(-\frac{4}{11}\right) = \frac{16}{11}$

The solution is $\left(-\frac{4}{11}, \frac{16}{11}\right)$.

My Notes

YOUR TURN

8. Ace Car Rental rents cars for x dollars per day plus y dollars for each mile driven. Carlos rented a car for 4 days, drove it 160 miles, and spent $120. Vanessa rented a car for 1 day, drove it 240 miles, and spent $80. Write equations to represent Carlos's expenses and Vanessa's expenses. Then solve the system and tell what each number represents.

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Guided Practice

Solve each system of linear equations by substitution. (Example 1)

1. $\begin{cases} 3x - 2y = 9 \\ y = 2x - 7 \end{cases}$ ____________

2. $\begin{cases} y = x - 4 \\ 2x + y = 5 \end{cases}$ ____________

3. $\begin{cases} x + 4y = 6 \\ y = -x + 3 \end{cases}$ ____________

4. $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$ ____________

Solve each system. Estimate the solution first. (Example 2)

5. $\begin{cases} 6x + y = 4 \\ x - 4y = 19 \end{cases}$

Estimate ____________

Solution ____________

6. $\begin{cases} x + 2y = 8 \\ 3x + 2y = 6 \end{cases}$

Estimate ____________

Solution ____________

7. $\begin{cases} 3x + y = 4 \\ 5x - y = 22 \end{cases}$

Estimate ____________

Solution ____________

8. $\begin{cases} 2x + 7y = 2 \\ x + y = -1 \end{cases}$

Estimate ____________

Solution ____________

9. Adult tickets to Space City amusement park cost x dollars. Children's tickets cost y dollars. The Henson family bought 3 adult and 1 child tickets for \$163. The Garcia family bought 2 adult and 3 child tickets for \$174. (Example 3)

a. Write equations to represent the Hensons' cost and the Garcias' cost.

Hensons' cost: ____________ Garcias' cost: ____________

b. Solve the system.

adult ticket price: ____________ child ticket price: ____________

ESSENTIAL QUESTION CHECK-IN

10. How can you decide which variable to solve for first when you are solving a linear system by substitution?

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Name ______________________ Class ______________ Date ______________

8.2 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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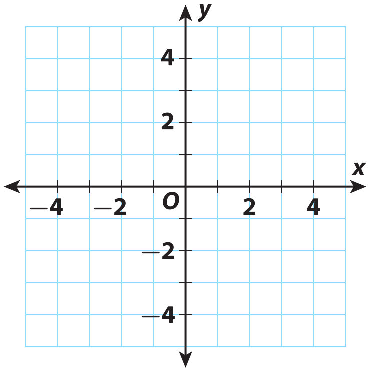
11. **Check for Reasonableness** Zach solves the system $\begin{cases} x + y = -3 \\ x - y = 1 \end{cases}$ and finds the solution $(1, -2)$. Use a graph to explain whether Zach's solution is reasonable.



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12. **Represent Real-World Problems** Angelo bought apples and bananas at the fruit stand. He bought 20 pieces of fruit and spent \$11.50. Apples cost \$0.50 and bananas cost \$0.75 each.

a. Write a system of equations to model the problem. (Hint: One equation will represent the number of pieces of fruit. A second equation will represent the money spent on the fruit.)

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b. Solve the system algebraically. Tell how many apples and bananas Angelo bought.

____

13. **Represent Real-World Problems** A jar contains n nickels and d dimes. There is a total of 200 coins in the jar. The value of the coins is \$14.00. How many nickels and how many dimes are in the jar?

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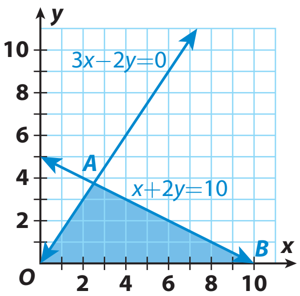
14. **Multistep** The graph shows a triangle formed by the x-axis, the line $3x - 2y = 0$, and the line $x + 2y = 10$. Follow these steps to find the area of the triangle.



a. Find the coordinates of point A by solving the system $\begin{cases} 3x - 2y = 0 \\ x + 2y = 10 \end{cases}$.

Point A: ______________________________

b. Use the coordinates of point A to find the height of the triangle.

height: ______________________________

c. What is the length of the base of the triangle?

base: ______________________________

d. What is the area of the triangle? ______________________________

15. Jed is graphing the design for a kite on a coordinate grid. The four vertices of the kite are at $A\left(-\frac{4}{3}, \frac{2}{3}\right)$, $B\left(\frac{14}{3}, -\frac{4}{3}\right)$, $C\left(\frac{14}{3}, -\frac{16}{3}\right)$, and $D\left(\frac{2}{3}, -\frac{16}{3}\right)$. One kite strut will connect points A and C. The other will connect points B and D. Find the point where the struts cross.

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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. **Analyze Relationships** Consider the system $\begin{cases} 6x - 3y = 15 \\ x + 3y = -8 \end{cases}$. Describe three different substitution methods that can be used to solve this system. Then solve the system.

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17. **Communicate Mathematical Ideas** Explain the advantages, if any, that solving a system of linear equations by substitution has over solving the same system by graphing.

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18. **Persevere in Problem Solving** Create a system of equations of the form $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$ that has $(7, -2)$ as its solution. Explain how you found the system.

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LESSON 8.3

Solving Systems by Elimination

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION

How do you solve a system of linear equations by adding or subtracting?

Solving a Linear System by Adding

The **elimination method** is another method used to solve a system of linear equations. In this method, one variable is *eliminated* by adding or subtracting the two equations of the system to obtain a single equation in one variable. The steps for this method are as follows:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value into either original equation to find the value of the eliminated variable.

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EXAMPLE 1

CA CC 8.EE.8b

Solve the system of equations by adding. Check your answer.

$$\begin{cases} 2x - 3y = 12 \\ x + 3y = 6 \end{cases}$$

STEP 1 Add the equations.

$2x - 3y = 12$ Write the equations so that like terms are aligned.

$+ (x + 3y = 6)$ Notice that the terms $-3y$ and $3y$ are opposites.

$3x + 0 = 18$ Add to eliminate the variable y.

$3x = 18$ Simplify and solve for x.

$\frac{3x}{3} = \frac{18}{3}$ Divide each side by 3.

$x = 6$ Simplify.

STEP 2 Substitute the solution into one of the original equations and solve for y.

$x + 3y = 6$ Use the second equation.

$6 + 3y = 6$ Substitute 6 for the variable x.

$3y = 0$ Subtract 6 from each side.

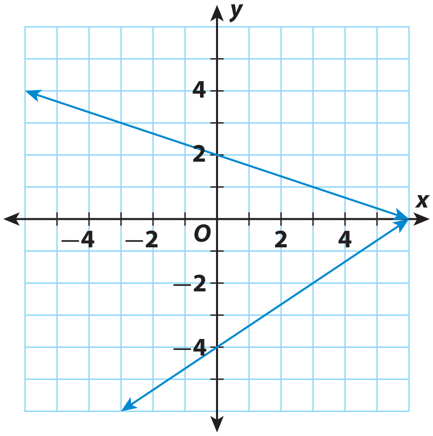
$y = 0$ Divide each side by 3 and simplify.

My Notes

STEP 3 Write the solution as an ordered pair: (6, 0)

STEP 4 Check the solution by graphing.

$2x - 3y = 12$	$x + 3y = 6$
x-intercept: 6	x-intercept: 6
y-intercept: -4	y-intercept: 2



The point of intersection is (6, 0).

Math Talk
Mathematical Practices

Is it better to check a solution by graphing or by substituting the values in the original equations?

Reflect

1. Can this linear system be solved by subtracting one of the original equations from the other? Why or why not?

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2. What is another way to check your solution?

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YOUR TURN

Solve each system of equations by adding. Check your answers.

3. $\begin{cases} x + y = -1 \\ x - y = 7 \end{cases}$ ____________

4. $\begin{cases} 2x + 2y = -2 \\ 3x - 2y = 12 \end{cases}$ ____________

5. $\begin{cases} 6x + 5y = 4 \\ -6x + 7y = 20 \end{cases}$ ____________

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Solving a Linear System by Subtracting

If both equations contain the same x- or y-term, you can solve by subtracting.

EXAMPLE 2

CA CC 8.EE.8b

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Solve the system of equations by subtracting. Check your answer.

$$\begin{cases} 3x + 3y = 6 \\ 3x - y = -6 \end{cases}$$

STEP 1 Subtract the equations.

$3x + 3y = 6$	Write the equations so that like terms are aligned.
$-(3x - y = -6)$	Notice that both equations contain the term $3x$.
$0 + 4y = 12$	Subtract to eliminate the variable x.
$4y = 12$	Simplify and solve for y.
$y = 3$	Divide each side by 4 and simplify.

STEP 2 Substitute the solution into one of the original equations and solve for x.

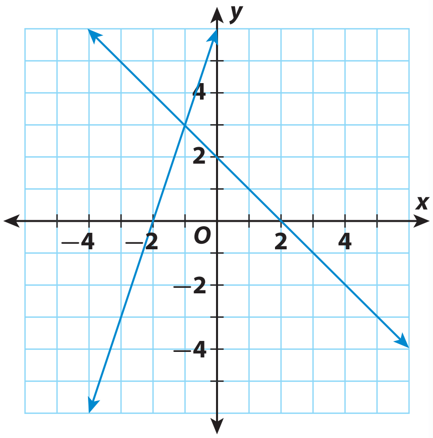
$3x - y = -6$	Use the second equation.
$3x - 3 = -6$	Substitute 3 for the variable y.
$3x = -3$	Add 3 to each side.
$x = -1$	Divide each side by 3 and simplify.

STEP 3 Write the solution as an ordered pair: $(-1, 3)$

STEP 4 Check the solution by graphing.

$3x + 3y = 6$	$3x - y = -6$
x-intercept: 2	x-intercept: -2
y-intercept: 2	y-intercept: 6

The point of intersection is $(-1, 3)$.



Reflect

6. **What If?** What would happen if you added the original equations?

My Notes

7. How can you decide whether to add or subtract to eliminate a variable in a linear system? Explain your reasoning.

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YOUR TURN

Solve each system of equations by subtracting. Check your answers.

8. $\begin{cases} 6x - 3y = 6 \\ 6x + 8y = -16 \end{cases}$

9. $\begin{cases} 4x + 3y = 19 \\ 6x + 3y = 33 \end{cases}$

10. $\begin{cases} 2x + 6y = 17 \\ 2x - 10y = 9 \end{cases}$

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Solving Problems with Systems of Equations

Many real-world situations can be modeled and solved with a system of equations.

EXAMPLE 3 Real World

CA CC 8.EE.8c

The Polar Bear Club wants to buy snowshoes and camp stoves. The club will spend \$554.50 to buy them at Top Sports and \$602.00 to buy them at Outdoor Explorer, before taxes, but Top Sports is farther away. How many of each item does the club intend to buy?

	Snowshoes	Camp Stoves
Top Sports	\$79.50 per pair	\$39.25
Outdoor Explorer	\$89.00 per pair	\$39.25

STEP 1 Choose variables and write a system of equations.
Let x represent the number of pairs of snowshoes.
Let y represent the number of camp stoves.

Top Sports cost: $79.50x + 39.25y = 554.50$
Outdoor Explorer cost: $89.00x + 39.25y = 602.00$

STEP 2 Subtract the equations.

$$\begin{array}{rl} 79.50x + 39.25y &= 554.50 \\ -(89.00x + 39.25y &= 602.00) \\ \hline -9.50x + 0 &= -47.50 \end{array}$$

Both equations contain the term 39.25y.

Subtract to eliminate the variable y.

$$-9.50x = -47.50$$ *Simplify and solve for x.*

$$\frac{-9.50x}{-9.50} = \frac{-47.50}{-9.50}$$ *Divide each side by −9.50.*

$$x = 5$$ *Simplify.*

STEP 3 Substitute the solution into one of the original equations and solve for y.

$$79.50x + 39.25y = 554.50$$ *Use the first equation.*

$$79.50(5) + 39.25y = 554.50$$ *Substitute 5 for the variable x.*

$$397.50 + 39.25y = 554.50$$ *Multiply.*

$$39.25y = 157.00$$ *Subtract 397.50 from each side.*

$$\frac{39.25y}{39.25} = \frac{157.00}{39.25}$$ *Divide each side by 39.25.*

$$y = 4$$ *Simplify.*

STEP 4 Write the solution as an ordered pair: (5, 4)

The club intends to buy 5 pairs of snowshoes and 4 camp stoves.

My Notes

YOUR TURN

11. At the county fair, the Baxter family bought 6 bags of roasted almonds and 4 juice drinks for \$16.70. The Farley family bought 3 bags of roasted almonds and 4 juice drinks for \$10.85. Find the price of a bag of roasted almonds and the price of a juice drink.

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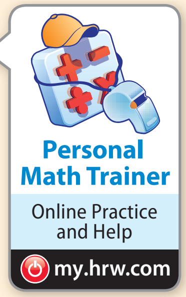

Guided Practice

1. Solve the system $\begin{cases} 4x + 3y = 1 \\ x - 3y = -11 \end{cases}$ by adding. (Example 1)

STEP 1 Add the equations.

$$\begin{array}{rl} 4x + 3y = 1 & \\ + \quad x - 3y = -11 & \\ \hline 5x + \boxed{} = \boxed{} & \end{array}$$

Write the equations so that like terms are aligned.

Add to eliminate the variable $\boxed{}$.

$5x = \boxed{}$ Simplify and solve for x.

$x = \boxed{}$ Divide both sides by $\boxed{}$ and simplify.

STEP 2 Substitute into one of the original equations and solve for y.

$y = \boxed{}$ So, $\boxed{}$ is the solution of the system.

Solve each system of equations by adding or subtracting. (Examples 1, 2)

2. $\begin{cases} x + 2y = -2 \\ -3x + 2y = -10 \end{cases}$

____

3. $\begin{cases} 3x + y = 23 \\ 3x - 2y = 8 \end{cases}$

____

4. $\begin{cases} -4x - 5y = 7 \\ 3x + 5y = -14 \end{cases}$

____

5. $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$

____

6. $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$

____

7. $\begin{cases} -5x + 7y = 11 \\ -5x + 3y = 19 \end{cases}$

____

8. The Green River Freeway has a minimum and a maximum speed limit. Tony drove for 2 hours at the minimum speed limit and 3.5 hours at the maximum limit, a distance of 355 miles. Rae drove 2 hours at the minimum speed limit and 3 hours at the maximum limit, a distance of 320 miles. What are the two speed limits? (Example 3)

a. Write equations to represent Tony's distance and Rae's distance.

Tony: ______________ Rae: ______________

b. Solve the system.

minimum speed limit: ________ maximum speed limit: ________

ESSENTIAL QUESTION CHECK-IN

9. Can you use addition or subtraction to solve any system? Explain.

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Name ______________________ Class ______________ Date ____________

8.3 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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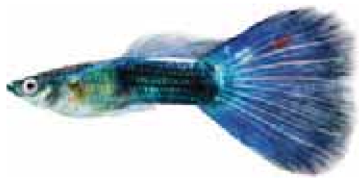
10. **Represent Real-World Problems** Marta bought new fish for her home aquarium. She bought 3 guppies and 2 platies for a total of \$13.95. Hank also bought guppies and platies for his aquarium. He bought 3 guppies and 4 platies for a total of \$18.33. Find the price of a guppy and the price of a platy.



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11. **Represent Real-World Problems** The rule for the number of fish in a home aquarium is 1 gallon of water for each inch of fish length. Marta's aquarium holds 13 gallons and Hank's aquarium holds 17 gallons. Based on the number of fish they bought in Exercise 10, how long is a guppy and how long is a platy?

____

12. Line m passes through the points (6, 1) and (2, -3). Line n passes through the points (2, 3) and (5, -6). Find the point of intersection of these lines.

____

13. **Represent Real-World Problems** Two cars got an oil change at the same auto shop. The shop charges customers for each quart of oil plus a flat fee for labor. The oil change for one car required 5 quarts of oil and cost \$22.45. The oil change for the other car required 7 quarts of oil and cost \$25.45. How much is the labor fee and how much is each quart of oil?

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14. **Represent Real-World Problems** A sales manager noticed that the number of units sold for two T-shirt styles, style A and style B, was the same during June and July. In June, total sales were \$2779 for the two styles, with A selling for \$15.95 per shirt and B selling for \$22.95 per shirt. In July, total sales for the two styles were \$2385.10, with A selling at the same price and B selling at a discount of 22% off the June price. How many T-shirts of each style were sold in June and July combined?

____

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15. **Represent Real-World Problems** Adult tickets to a basketball game cost \$5. Student tickets cost \$1. A total of \$2,874 was collected on the sale of 1,246 tickets. How many of each type of ticket were sold?

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. **Communicate Mathematical Ideas** Is it possible to solve the system $\begin{cases} 3x - 2y = 10 \\ x + 2y = 6 \end{cases}$ by using substitution? If so, explain how. Which method, substitution or elimination, is more efficient? Why?

17. Jenny used substitution to solve the system $\begin{cases} 2x + y = 8 \\ x - y = 1 \end{cases}$. Her solution is shown below.

Step 1	$y = -2x + 8$	Solve the first equation for y.
Step 2	$2x + (-2x + 8) = 8$	Substitute the value of y in an original equation.
Step 3	$2x - 2x + 8 = 8$	Use the Distributive Property.
Step 4	$8 = 8$	Simplify.

a. **Explain the Error** Explain the error Jenny made. Describe how to correct it.

b. **Communicate Mathematical Ideas** Would adding the equations have been a better method for solving the system? If so, explain why.

LESSON 8.4

Solving Systems by Elimination with Multiplication

CA CC 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

ESSENTIAL QUESTION

How do you solve a system of linear equations by multiplying?

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Solving a System by Multiplying and Adding

In some linear systems, neither variable can be eliminated by adding or subtracting the equations directly. In systems like these, you need to multiply one of the equations by a constant so that adding or subtracting the equations will eliminate one variable. The steps for this method are as follows:

1. Decide which variable to eliminate.
2. Multiply one equation by a constant so that adding or subtracting will eliminate that variable.
3. Solve the system using the elimination method.

EXAMPLE 1

CA CC 8.EE.8b

Solve the system of equations by multiplying and adding.

$$\begin{cases} 2x + 10y = 2 \\ 3x - 5y = -17 \end{cases}$$

STEP 1 The coefficient of y in the first equation, 10, is 2 times the coefficient of y, 5, in the second equation. Also, the y-term in the first equation is being added, while the y-term in the second equation is being subtracted. To eliminate the y-terms, multiply the second equation by 2 and add this new equation to the first equation.

$2(3x - 5y = -17)$ — *Multiply each term in the second equation by 2 to get opposite coefficients for the y-terms.*

$6x - 10y = -34$ — *Simplify.*

$$\begin{array}{r} 6x - 10y = -34 \\ +\ 2x + 10y = 2 \\ \hline 8x + 0y = -32 \end{array}$$

Add the first equation to the new equation.

Add to eliminate the variable y.

$8x = -32$ — *Simplify and solve for x.*

$\frac{8x}{8} = \frac{-32}{8}$ — *Divide each side by 8.*

$x = -4$ — *Simplify.*

My Notes

STEP 2 Substitute the solution into one of the original equations and solve for *y*.

$$\begin{aligned} 2x + 10y &= 2 && \text{Use the first equation.} \\ 2(-4) + 10y &= 2 && \text{Substitute } -4 \text{ for the variable } x. \\ -8 + 10y &= 2 && \text{Simplify.} \\ 10y &= 10 && \text{Add 8 to each side.} \\ y &= 1 && \text{Divide each side by 10 and simplify.} \end{aligned}$$

Math Talk
Mathematical Practices

When you check your answer algebraically, why do you substitute your values for *x* and y into the original system? Explain.

STEP 3 Write the solution as an ordered pair: $(-4, 1)$

STEP 4 Check your answer algebraically.

Substitute -4 for *x* and 1 for *y* in the original system.

$$\begin{cases} 2x + 10y = 2 \rightarrow 2(-4) + 10(1) = -8 + 10 = 2 \checkmark \\ 3x - 5y = -17 \rightarrow 3(-4) - 5(1) = -12 - 5 = -17 \checkmark \end{cases}$$

The solution is correct.

Reflect

1. How can you solve this linear system by subtracting? Which is more efficient, adding or subtracting? Explain your reasoning.

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2. Can this linear system be solved by adding or subtracting without multiplying? Why or why not?

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3. What would you need to multiply the second equation by to eliminate *x* by adding? Why might you choose to eliminate *y* instead of *x*?

____

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YOUR TURN

Solve each system of equations by multiplying and adding.

4. $\begin{cases} 5x + 2y = -10 \\ 3x + 6y = 66 \end{cases}$ _______________

5. $\begin{cases} 4x + 2y = 6 \\ 3x - y = -8 \end{cases}$ _______________

6. $\begin{cases} -6x + 9y = -12 \\ 2x + y = 0 \end{cases}$ _______________

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Solving a System by Multiplying and Subtracting

You can solve some systems of equations by multiplying one equation by a constant and then subtracting.

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EXAMPLE 2

CA CC 8.EE.8b

Solve the system of equations by multiplying and subtracting.

$$\begin{cases} 6x + 5y = 7 \\ 2x - 4y = -26 \end{cases}$$

STEP 1 Multiply the second equation by 3 and subtract this new equation from the first equation.

$3(2x - 4y = -26)$	*Multiply each term in the second equation by 3 to get the same coefficients for the x-terms.*
$6x - 12y = -78$	*Simplify.*
$6x + 5y = 7$ $-(6x - 12y = -78)$	*Subtract the new equation from the first equation.*
$0x + 17y = 85$	*Subtract to eliminate the variable x.*
$17y = 85$	*Simplify and solve for y.*
$\frac{17y}{17} = \frac{85}{17}$	*Divide each side by 17.*
$y = 5$	*Simplify.*

STEP 2 Substitute the solution into one of the original equations and solve for x.

$6x + 5y = 7$	*Use the first equation.*
$6x + 5(5) = 7$	*Substitute 5 for the variable y.*
$6x + 25 = 7$	*Simplify.*
$6x = -18$	*Subtract 25 from each side.*
$x = -3$	*Divide each side by 6 and simplify.*

STEP 3 Write the solution as an ordered pair: $(-3, 5)$

STEP 4 Check your answer algebraically.
Substitute -3 for x and 5 for y in the original system.

$$\begin{cases} 6x + 5y = 7 \rightarrow 6(-3) + 5(5) = -18 + 25 = 7 \checkmark \\ 2x - 4y = -26 \rightarrow 2(-3) - 4(5) = -6 - 20 = -26 \checkmark \end{cases}$$

The solution is correct.

My Notes

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YOUR TURN

Solve each system of equations by multiplying and subtracting.

7. $\begin{cases} 3x - 7y = 2 \\ 6x - 9y = 9 \end{cases}$ ____________

8. $\begin{cases} -3x + y = 11 \\ 2x + 3y = -11 \end{cases}$ ____________

9. $\begin{cases} 9x + y = 9 \\ 3x - 2y = -11 \end{cases}$ ____________

Solving Problems with Systems of Equations

Many real-world situations can be modeled with a system of equations.

EXAMPLE 3 Problem Solving

CA CC 8.EE.8c

The Simon family attended a concert and visited an art museum. Concert tickets were \$24.75 for adults and \$16.00 for children, for a total cost of \$138.25. Museum tickets were \$8.25 for adults and \$4.50 for children, for a total cost of \$42.75. How many adults and how many children are in the Simon family?

Analyze Information

The answer is the number of adults and children.

Formulate a Plan

Solve a system to find the number of adults and children.

Solve

STEP 1 Choose variables and write a system of equations. Let x represent the number of adults. Let y represent the number of children.

Concert cost: $24.75x + 16.00y = 138.25$
Museum cost: $8.25x + 4.50y = 42.75$

STEP 2 Multiply both equations by 100 to eliminate the decimals.

$100(24.75x + 16.00y = 138.25) \rightarrow 2{,}475x + 1{,}600y = 13{,}825$

$100(8.25x + 4.50y = 42.75) \rightarrow 825x + 450y = 4{,}275$

STEP 3 Multiply the second equation by 3 and subtract this new equation from the first equation.

$$3(825x + 450y = 4{,}275)$$

Multiply each term in the second equation by 3 to get the same coefficients for the x-terms.

$$2{,}475x + 1{,}350y = 12{,}825$$

Simplify.

$$\begin{array}{r} 2{,}475x + 1{,}600y = 13{,}825 \\ -(2{,}475x + 1{,}350y = 12{,}825) \\ \hline 0x + 250y = 1{,}000 \end{array}$$

Subtract the new equation from the first equation.

Subtract to eliminate the variable x.

$$250y = 1{,}000$$

Simplify and solve for y.

$$\frac{250y}{250} = \frac{1{,}000}{250}$$

Divide each side by 250.

$$y = 4$$

Simplify.

STEP 4 Substitute the solution into one of the original equations and solve for x.

$$8.25x + 4.50y = 42.75$$

Use the second equation.

$$8.25x + 4.50(4) = 42.75$$

Substitute 4 for the variable y.

$$8.25x + 18 = 42.75$$

Simplify.

$$8.25x = 24.75$$

Subtract 18 from each side.

$$x = 3$$

Divide each side by 8.25 and simplify.

STEP 5 Write the solution as an ordered pair: (3, 4).
There are 3 adults and 4 children in the family.

Justify and Evaluate

Substituting $x = 3$ and $y = 4$ into the original equations results in true statements. The answer is correct.

My Notes

YOUR TURN

10. Contestants in the Run-and-Bike-a-thon run for a specified length of time, then bike for a specified length of time. Jason ran at an average speed of 5.2 mi/h and biked at an average speed of 20.6 mi/h, going a total of 14.2 miles. Seth ran at an average speed of 10.4 mi/h and biked at an average speed of 18.4 mi/h, going a total of 17 miles. For how long do contestants run and for how long do they bike?

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Guided Practice

1. Solve the system $\begin{cases} 3x - y = 8 \\ -2x + 4y = -12 \end{cases}$ by multiplying and adding. (Example 1)

STEP 1 Multiply the first equation by 4. Add to the second equation.

$4(3x - y = 8)$ — Multiply each term in the first equation by 4 to get opposite coefficients for the y-terms.

$\boxed{}x - \boxed{}y = \boxed{}$ — Simplify.

$+\ (-2x) + 4y = -12$ — Add the second equation to the new equation.

$10x = \boxed{}$ — Add to eliminate the variable $\boxed{}$.

$x = \boxed{}$ — Divide both sides by $\boxed{}$ and simplify.

STEP 2 Substitute into one of the original equations and solve for y.

$y = \boxed{}$ So, $\boxed{}$ is the solution of the system.

Solve each system of equations by multiplying first. (Examples 1, 2)

2. $\begin{cases} x + 4y = 2 \\ 2x + 5y = 7 \end{cases}$ ____________

3. $\begin{cases} 3x + y = -1 \\ 2x + 3y = 18 \end{cases}$ ____________

4. $\begin{cases} 2x + 8y = 21 \\ 6x - 4y = 14 \end{cases}$ ____________

5. $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$ ____________

6. $\begin{cases} 6x + 5y = 19 \\ 2x + 3y = 5 \end{cases}$ ____________

7. $\begin{cases} 2x + 5y = 16 \\ -4x + 3y = 20 \end{cases}$ ____________

8. Bryce spent \$5.26 on some apples priced at \$0.64 each and some pears priced at \$0.45 each. At another store he could have bought the same number of apples at \$0.32 each and the same number of pears at \$0.39 each, for a total cost of \$3.62. How many apples and how many pears did Bryce buy? (Example 3)

a. Write equations to represent Bryce's expenditures at each store.

First store: ____________________ Second store: ____________________

b. Solve the system.

Number of apples: ______________ Number of pears: ________________

ESSENTIAL QUESTION CHECK-IN

9. When solving a system by multiplying and then adding or subtracting, how do you decide whether to add or subtract?

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Name ____________________ Class ____________ Date ____________

8.4 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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10. **Explain the Error** Gwen used elimination with multiplication to solve the system $\begin{cases} 2x + 6y = 3 \\ x - 3y = -1 \end{cases}$. Her work to find x is shown. Explain her error. Then solve the system.

$$\begin{aligned} 2(x - 3y) &= -1 \\ 2x - 6y &= -1 \\ \underline{+2x + 6y} &\underline{= 3} \\ 4x + 0y &= 2 \\ x &= \frac{1}{2} \end{aligned}$$

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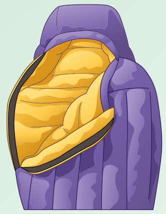
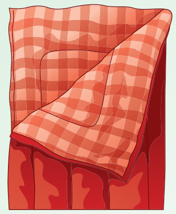
11. **Represent Real-World Problems** At Raging River Sports, polyester-fill sleeping bags sell for \$79. Down-fill sleeping bags sell for \$149. In one week the store sold 14 sleeping bags for \$1456.

Sleeping Bags



Nylon
Down-filled, 35°
\$149



Flannel-lined
Polyester-filled, 40°
\$79

a. Let x represent the number of polyester-fill bags sold and let y represent the number of down-fill bags sold. Write a system of equations you can solve to find the number of each type sold.

____

b. Explain how you can solve the system for y by multiplying and subtracting.

____

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c. Explain how you can solve the system for y using substitution.

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d. How many of each type of bag were sold?

____

12. Twice a number plus twice a second number is 310. The difference between the numbers is 55. Find the numbers by writing and solving a system of equations. Explain how you solved the system.

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13. **Represent Real-World Problems** A farm stand sells apple pies and jars of applesauce. The table shows the number of apples needed to make a pie and a jar of applesauce. Yesterday, the farm picked 169 Granny Smith apples and 95 Golden Delicious apples. How many pies and jars of applesauce can the farm make if every apple is used?

Type of apple	Granny Smith	Golden Delicious
Needed for a pie	5	3
Needed for a jar of applesauce	4	2

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

14. **Make a Conjecture** Lena tried to solve a system of linear equations algebraically and in the process found the equation $5 = 9$. Lena thought something was wrong, so she graphed the equations and found that they were parallel lines. Explain what Lena's graph and equation could mean.

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15. Consider the system $\begin{cases} 2x + 3y = 6 \\ 3x + 7y = -1 \end{cases}$.

a. **Communicate Mathematical Ideas** Describe how to solve the system by multiplying the first equation by a constant and subtracting. Why would this method be less than ideal?

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b. **Draw Conclusions** Is it possible to solve the system by multiplying both equations by integer constants? If so, explain how.

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c. Use your answer from part b to solve the system.

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LESSON 8.5

Solving Special Systems

CA CC 8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *Also 8.EE.8c*

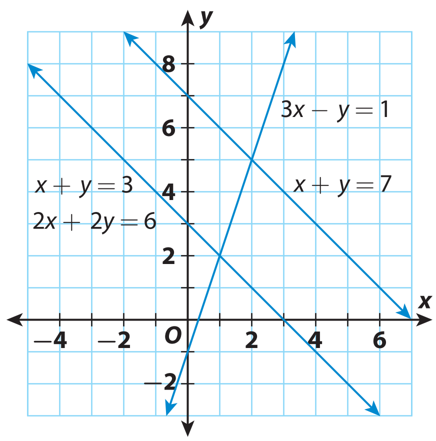
ESSENTIAL QUESTION

How do you solve systems with no solution or infinitely many solutions?

EXPLORE ACTIVITY CA CC 8.EE.8b

Solving Special Systems by Graphing

As with linear equations in one variable, some systems may have no solution or infinitely many solutions. One way to tell how many solutions a system has is by inspecting its graph.



Use the graph to solve each system of linear equations.

A $\begin{cases} x + y = 7 \\ 2x + 2y = 6 \end{cases}$

Is there a point of intersection? Explain.

____

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Does this linear system have a solution? Use the graph to explain.

____

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B $\begin{cases} 2x + 2y = 6 \\ x + y = 3 \end{cases}$

Is there a point of intersection? Explain.

____

Does this linear system have a solution? Use the graph to explain.

____

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Reflect

1. **Justify Reasoning** Use the graph to identify two lines that represent a linear system with exactly one solution. What are the equations of the lines? Explain your reasoning.

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EXPLORE ACTIVITY *(cont'd)*

2. A system of linear equations has infinitely many solutions. Does that mean any ordered pair in the coordinate plane is a solution?

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3. Identify the three possible numbers of solutions for a system of linear equations. Explain when each type of solution occurs.

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Solving Special Systems Algebraically

As with equations, if you solve a system of equations with no solution, you get a false statement, and if you solve a system with infinitely many solutions, you get a true statement.

EXAMPLE 1

CA CC 8.EE.8b

A Solve the system of linear equations by substitution.

$$\begin{cases} x - y = -2 \\ -x + y = 4 \end{cases}$$

STEP 1 Solve $x - y = -2$ for x:

$x = y - 2$

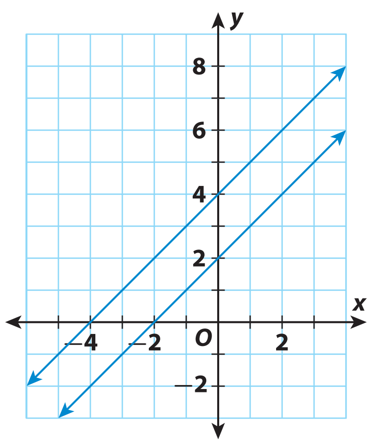
STEP 2 Substitute the resulting expression into the other equation and solve.

$-(y - 2) + y = 4$ Substitute the expression for the variable x.

$2 = 4$ Simplify.

STEP 3 Interpret the solution. The result is the false statement $2 = 4$, which means there is no solution.

STEP 4 Graph the equations to check your answer. The graphs do not intersect, so there is no solution.



Math Talk
Mathematical Practices

The system can be written using $-x + y = 2$ and $-x + y = 4$. How does this show you there is no solution just by looking at it?

B Solve the system of linear equations by elimination.

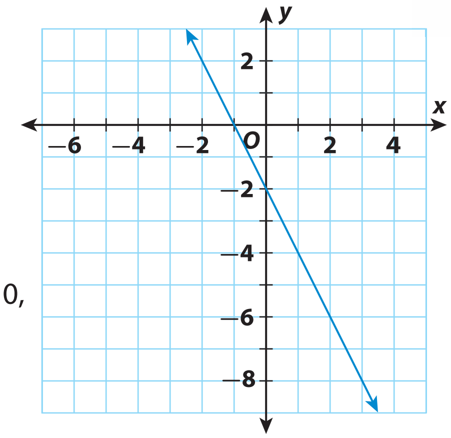
$$\begin{cases} 2x + y = -2 \\ 4x + 2y = -4 \end{cases}$$

STEP 1 Multiply the first equation by -2.

$-2(2x + y = -2) \rightarrow -4x + (-2y) = 4$

STEP 2 Add the new equation from Step 1 to the original second equation.

$$\begin{array}{rrcr} & -4x + (-2y) & = & 4 \\ + & 4x + 2y & = & -4 \\ \hline & 0x + 0y & = & 0 \\ & 0 & = & 0 \end{array}$$



STEP 3 Interpret the solution. The result is the statement $0 = 0$, which is always true. This means that the system has infinitely many solutions.

STEP 4 Graph the equations to check your answer. The graphs are the same line, so there are infinitely many solutions.

Math Talk
Mathematical Practices

What solution do you get when you solve the system in part B by substitution? Does this result change the number of solutions? Explain.

Reflect

4. If x represents a variable and a and b represent constants so that $a \neq b$, interpret what each result means when solving a system of equations.

$x = a$ ______________________________

$a = b$ ______________________________

$a = a$ ______________________________

5. **Draw Conclusions** In part B, can you tell without solving that the system has infinitely many solutions? If so, how?

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YOUR TURN

Solve each system. Tell how many solutions each system has.

6. $\begin{cases} 4x - 6y = 9 \\ -2x + 3y = 4 \end{cases}$

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7. $\begin{cases} x + 2y = 6 \\ 2x - 3y = 26 \end{cases}$

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8. $\begin{cases} 12x - 8y = -4 \\ -3x + 2y = 1 \end{cases}$

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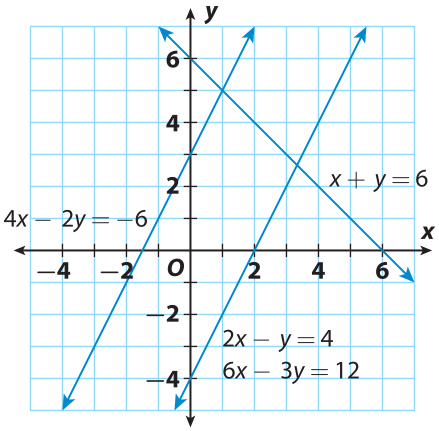
Guided Practice

1. Use the graph to find the number of solutions of each system of linear equations. (Explore Activity)

A. $\begin{cases} 4x - 2y = -6 \\ 2x - y = 4 \end{cases}$ **B.** $\begin{cases} 4x - 2y = -6 \\ x + y = 6 \end{cases}$ **C.** $\begin{cases} 2x - y = 4 \\ 6x - 3y = 12 \end{cases}$



STEP 1 Decide if the graphs of the equations in each system intersect, are parallel, or are the same line.

System A: The graphs ______________________ .

System B: The graphs ______________________ .

System C: The graphs ______________________ .

STEP 2 Use the results of Step 1 to decide how many points the graphs have in common.

System A has ______________________ point(s) in common.

System B has ______________________ point(s) in common.

System C has ______________________ point(s) in common.

Solve each system. Tell how many solutions each system has. (Example 1)

2. $\begin{cases} x - 3y = 4 \\ -5x + 15y = -20 \end{cases}$

____

3. $\begin{cases} 6x + 2y = -4 \\ 3x + y = 4 \end{cases}$

____

4. $\begin{cases} 6x - 2y = -10 \\ 3x + 4y = -25 \end{cases}$

____

Find the number of solutions of each system without solving algebraically or graphing. Explain your reasoning. (Example 1)

5. $\begin{cases} -6x + 2y = -8 \\ 3x - y = 4 \end{cases}$

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6. $\begin{cases} 8 = 7x + 2y \\ 7x + 2y = 4 \end{cases}$

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ESSENTIAL QUESTION CHECK-IN

7. When you solve a system of equations algebraically, how can you tell whether the system has zero, one, or an infinite number of solutions?

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Name ____________ Class ____________ Date ____________

8.5 Independent Practice

CA CC 8.EE.8b, 8.EE.8c

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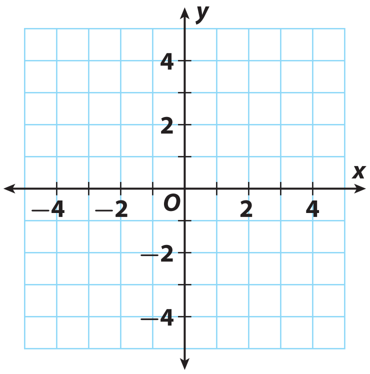
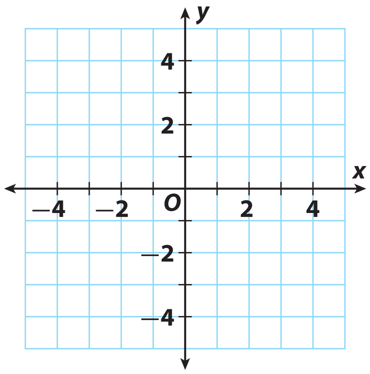
Solve each system by graphing. Check your answer algebraically.

8. $\begin{cases} -2x + 6y = 12 \\ x - 3y = 3 \end{cases}$



Solution: ____________

9. $\begin{cases} 15x + 5y = 5 \\ 3x + y = 1 \end{cases}$



Solution: ____________

For Exs. 10–16, state the number of solutions for each system of linear equations.

10. a system whose graphs have the same slope but different *y*-intercepts

____

11. a system whose graphs have the same *y*-intercepts but different slopes

____

12. a system whose graphs have the same *y*-intercepts and the same slopes

____

13. a system whose graphs have different *y*-intercepts and different slopes

____

14. the system $\begin{cases} y = 2 \\ y = -3 \end{cases}$ ____________

15. the system $\begin{cases} x = 2 \\ y = -3 \end{cases}$ ____________

16. the system whose graphs were drawn using these tables of values:

Equation 1

x	0	1	2	3
y	1	3	5	7

Equation 2

x	0	1	2	3
y	3	5	7	9

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17. **Draw Conclusions** The graph of a linear system appears in a textbook. You can see that the lines do not intersect on the graph, but also they do not appear to be parallel. Can you conclude that the system has no solution? Explain.

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18. **Represent Real-World Problems** Two school groups go to a roller skating rink. One group pays \$243 for 36 admissions and 21 skate rentals. The other group pays \$81 for 12 admissions and 7 skate rentals. Let x represent the cost of admission and let y represent the cost of a skate rental. Is there enough information to find values for x and y? Explain.

19. **Represent Real-World Problems** Juan and Tory are practicing for a track meet. They start their practice runs at the same point, but Tory starts 1 minute after Juan. Both run at a speed of 704 feet per minute. Does Tory catch up to Juan? Explain.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

20. **Justify Reasoning** A linear system with no solution consists of the equation $y = 4x - 3$ and a second equation of the form $y = mx + b$. What can you say about the values of m and b? Explain your reasoning.

21. **Justify Reasoning** A linear system with infinitely many solutions consists of the equation $3x + 5 = 8$ and a second equation of the form $Ax + By = C$. What can you say about the values of A, B, and C? Explain your reasoning.

22. **Draw Conclusions** Both the points $(2, -2)$ and $(4, -4)$ are solutions of a system of linear equations. What conclusions can you make about the equations and their graphs?

MODULE QUIZ

Ready to Go On?

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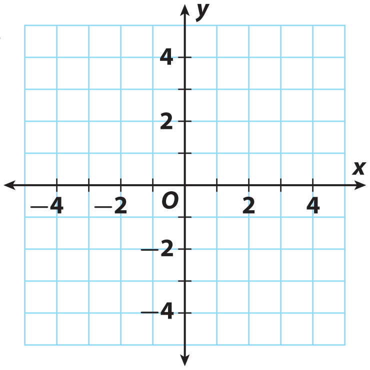
8.1 Solving Systems of Linear Equations by Graphing

Solve each system by graphing.

1. $\begin{cases} y = x - 1 \\ y = 2x - 3 \end{cases}$



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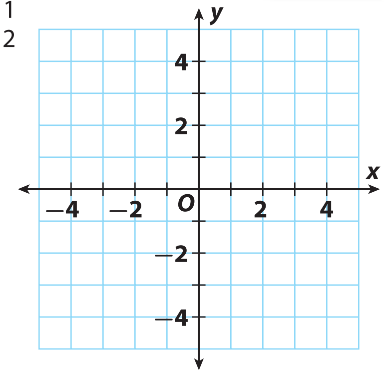
2. $\begin{cases} x + 2y = 1 \\ -x + y = 2 \end{cases}$



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8.2 Solving Systems by Substitution

Solve each system of equations by substitution.

3. $\begin{cases} y = 2x \\ x + y = -9 \end{cases}$ _______________

4. $\begin{cases} 3x - 2y = 11 \\ x + 2y = 9 \end{cases}$ _______________

8.3 Solving Systems by Elimination

Solve each system of equations by adding or subtracting.

5. $\begin{cases} 3x + y = 9 \\ 2x + y = 5 \end{cases}$ _______________

6. $\begin{cases} -x - 2y = 4 \\ 3x + 2y = 4 \end{cases}$ _______________

8.4 Solving Systems by Elimination with Multiplication

Solve each system of equations by multiplying first.

7. $\begin{cases} x + 3y = -2 \\ 3x + 4y = -1 \end{cases}$ _______________

8. $\begin{cases} 2x + 8y = 22 \\ 3x - 2y = 5 \end{cases}$ _______________

8.5 Solving Special Systems

Solve each system. Tell how many solutions each system has.

9. $\begin{cases} -2x + 8y = 5 \\ x - 4y = -3 \end{cases}$ _______________

10. $\begin{cases} 6x + 18y = -12 \\ x + 3y = -2 \end{cases}$ _______________

ESSENTIAL QUESTION

11. What are the possible solutions to a system of linear equations, and what do they represent graphically?

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MODULE 8
MIXED REVIEW

Assessment Readiness

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1. Consider each system of equations. Does the system have at least one solution? Select Yes or No for systems A–C.

A. $\begin{cases} 3x - 2y = 4 \\ -6x + 4y = -8 \end{cases}$ ◯ Yes ◯ No

B. $\begin{cases} -4x + y = 1 \\ 12x - 3y = 3 \end{cases}$ ◯ Yes ◯ No

C. $\begin{cases} 2x + y = 0 \\ 4x - y = -6 \end{cases}$ ◯ Yes ◯ No

2. Ruby ran 5 laps on the inside lane around a track. Dana ran 3 laps around the same lane on the same track, and then she ran 0.5 mile to her house. Both girls ran the same number of miles in all. The equation $5d = 3d + 0.5$ models this situation.

Choose True or False for each statement.

A. The variable d represents the distance around the track in miles. ◯ True ◯ False

B. The expression $5d$ represents the time it took Ruby to run 5 laps. ◯ True ◯ False

C. The expression $3d + 0.5$ represents the distance in miles Dana ran. ◯ True ◯ False

3. Daisies cost \$0.99 each and tulips cost \$1.15 each. Maria bought a bouquet of daisies and tulips. It contained 12 flowers and cost \$12.52. Solve the system $\begin{cases} x + y = 12 \\ 0.99x + 1.15y = 12.52 \end{cases}$ to find the number of daisies x and the number of tulips y in Maria's bouquet. State the method you used to solve the system and why you chose that method.

4. Kylie bought 4 daily bus passes and 2 weekly bus passes for \$29.00. Luis bought 7 daily bus passes and 2 weekly bus passes for \$36.50. Write and solve a system of equations to find the cost of a daily pass and the cost of a weekly pass. Explain how you can check your answer.